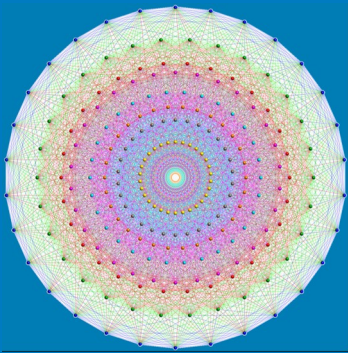




VisibLie_E₈

A Theory Of Everything Visualizer
aka. The HyperComplex HyperDimensional Polytope X-ray Machine

“Allegorically casting 3D shadows in Plato's 4D CAVE from an E₈³ 8D Universe using H₄ - the 4D analog of 3D Platonic solids, as derived from Pascal's Triangle while trying to understand the Codes of GR/QM and SM physics, Quantum chemistry, and the origins of Life from the genetic phyllotaxis of Fibonacci's sequence” - G. Moxness

“Algebra without geometry is blind, geometry without algebra is dumb” - D. Hestenes
“... both are incomplete and powerless without computation.” - N. Murzin

Greg Moxness
TheoryOfEverything.org

J Gregory Moxness’ (JGM) Independent Mathematica Analysis of Jenny Lorraine Nielsen’s (JLN) Topological Unified Field Theory (TUFT)

This Mathematica Notebook was written by J. Gregory Moxness <https://www.TheoryOfEverything.org/theToE>
This is an analysis of the 7/1/2026 (i.e. the PDF creation date - **not page 1 listed date**) version(s) of this paper.
Note: The single pdf has 2 version copies. The main “condensed” version ends at page 89
and the “dissertation version” (which is my reference for analysis) is appended below that to page 290:
It is a numeric analysis of my symbolic construction of the paper’s equations. My theoretical analysis is ongoing.
It uses both Wolfram’s SI units with their curated PDG quantities and experimental error values and my new “More Natural Units of Measure” (UoM).
These are based on my 2001 8D group-theoretic scaling of $c, \hbar, 1/G_N$, and $1/H_0$ to α^{-8} (in comparison to JLN α^{-16} of G_N).
JLN’s equation results are found to be (mostly) consistent with my symbolic / numerical analysis based on Wolfram curated PDG CODATA datasets.
JLN’s Paper Page 1:
In[5900]:=

The Topological Unified Field Theory on the Complex Hopf Fibration

*The Complex Hopf Fibration as the Canonical Space for Gauge–Gravity Unification:
The Field, Universal Action, and Particle Spectrum*

Jennifer “Jenny” Lorraine Nielsen*
Center for Topological Physics

;

June 18, 2026
(First Associated Preprint November 7, 2019; First Draft: February, 2023; Updated March 22, 2025; Submitted to IJT July 1, 2025;
Major Update September 17, 2025; Major Update October 26, 2025; Update January 29, 2026; Update February 8, 2026; Update
May 31, 2026; Major Update June 18, 2026; Passed Round 1 Peer Review; second Round Editors asked for shorter resubmission;
Passed third round; Revisions submitted 6/18/2026; still pending final decision
at *International Journal of Topology*)

“I want to know how God created this world.
I am not interested in this or that phenomenon ...
I want to know His thoughts; the rest are details.”
— Albert Einstein

JLN’s FQXI overview quip:
In[5901]:=

U(1) = S¹ fiber of complex Hopf
SU(2) = S³ total space of complex Hopf, with Chern Simons Gravity 1988
SU(3) = S⁵ total space of complex Hopf minus S³ embedded total space SU(2)
Bingo. There's the UFT! All on one field. (Quickest derivation yet ! Ha)
Philpapers.org/rec/NIETTU (Click Download PDF)
<https://www.preprints.org/manuscript/202604.0315> (Click download PDF)

;

Cosmological Parameters (G_N, v=VEV, H₀, Λ, Ω_{Λ=DE}, Ω_{DM}, Ω_m) Newton’s Gravitation Constant G_N:

JLN reference snapshots:
Section 7.3 page 224

In[5902]:=

Proof. The cocycle condition $g_{ij}g_{jk}g_{ki} = 1$ on triple overlaps requires $e^{2\pi i\alpha} = 1$, hence $\alpha \in \mathbb{Z}$. □

Corollary 24 (Angular Momentum Quantization). *The winding number $w \in \pi_1(S^1) \cong \mathbb{Z}$ of the horizontal lift of a closed loop in $\mathbb{CP}^4$ gives the eigenvalue of $\hat{L}_z = -i\hbar\partial_\phi$; periodicity of $e^{iw\phi}$ forces $L_z = \hbar w$ with $w \in \mathbb{Z}$.*

Definition 16 (Planck's Constant from the Fiber Cross-Section). *The S^1 fiber is the minimal-dimensional submanifold of the Hopf hierarchy. Its cross-sectional diameter—the physical width of the fiber considered as a tube—is the Planck length:*

$$\ell_P := \text{cross-sectional diameter of the } S^1 \text{ fiber.}$$

Since the speed of light c (Theorem 103) and Newton's constant G (Theorem 108) are both derived, Planck's reduced constant follows:

$$\hbar = \frac{\ell_P^2 c^3}{G} = 1.054571817 \times 10^{-34} \text{ J} \cdot \text{s}, \quad h = 2\pi\hbar.$$

Status. The identification of the fiber cross-section with ℓ_P is a definition: the S^1 fiber is the smallest geometric object in the Hopf hierarchy, and the Planck length is the smallest physical length. Given this identification, $\hbar$ is determined by two already-derived constants (c and G) and is not an independent input. Quantization arises because the fiber has a finite cross-section: only discrete modes fit inside a tube of finite cross-section.

In[5903]:=

(ii) Graviton coupling to all 16 subsectors. The graviton is the amphichiral figure-eight knot (4_1) mode on the S^1 fiber in the zero-winding ($n = 0$) sector. By amphichirality—existence of an orientation-reversing diffeomorphism of the total space compatible with the Hopf projection—this mode couples equally to all sectors of the coexact spectrum. No invariant substructure selects a proper subset of the 16 subsectors. The effective gravitational coupling is therefore the per-subsector coupling raised to the 16th power:

$$\alpha_G = \alpha^{16}.$$

The exponent 16 is a spectral fact about the contact distribution on S^9 , not a fitted integer.

(iii) Holonomy prefactor $(2\pi + \alpha)$. A graviton completing one S^1 circuit accumulates:

Geometric phase: 2π from the bare holonomy (Theorem 104).

Electromagnetic dressing: The graviton propagates in the background of the $U(1)$ connection. On the compact fiber S^1 , the one-loop vacuum polarization correction to the Wilson loop[92] evaluates to $\delta\mathcal{H} = \alpha$ (directly computable from the quadratic effective action on S^1 with discrete momentum spectrum $k_m = 2\pi mv$).

Total holonomy per circuit: $\mathcal{H} = 2\pi + \alpha$.

Assembly. By dimensional analysis, with v the sole dimensionful input (Axiom 2), Newton's constant must take the form $G = C \cdot \hbar c / v^2$, where C is built from the only available geometric invariants of the $n = 0$ graviton sector. The odd zeta values $\zeta(3), \zeta(5), \dots$ and the framing number $\ell = 6$ govern the massive ($n \geq 1$) sectors and do not enter the massless graviton coupling. The only dimensionless factors from the $n = 0$ sector are α^{16} (spectral subsector coupling) and $(2\pi + \alpha)$ (dressed holonomy). Therefore

$$G = (2\pi + \alpha) \alpha^{16} \frac{\hbar c}{v^2}.$$

□

JLN Supporting equations:

G_N curated Wolfram MTM data and 2024 PDG CODATA

In[5904]:=

GravitationalConstant

Out[5904]=

$$6.674 \times 10^{-11} \text{ Meter}^3$$

Kilogram Second²

In[5905]:=

NATURAL@% /. cRep

Out[5905]=

$$1. \text{ NaturalLengthUnit}^3$$

NaturalMassUnit NaturalTimeUnit²

$G_N \approx \alpha^8 M^{-1} L^3 T^{-2}$ from my new "More Natural" UoM based on 8 D geometric exponents for fine structure (α^{-8}).

In[5906]:=

UNITS@%

$$\% \frac{\alpha^{-8}}{g c^2} /. \text{cRep}$$

Out[5906]=

$$9.13592 \times 10^{-18} \text{ LengthUnit}^3$$

MassUnit TimeUnit²

Out[5907]=

$$0.999313 \text{ LengthUnit}^3$$

MassUnit TimeUnit²

We have the JLN G_N derivation as $\alpha^{16} \hbar c = 1 \text{ ML}^3 \text{ T}^{-2}$ in my UoM:

In[5908]:=

$$\alpha^{16} c \hbar /. \text{cRep}$$

Out[5908]=

$$1. \text{ LengthUnit}^3 \text{ MassUnit}$$

TimeUnit²

The square root of (dressed) Holonomy preFactor ($\sqrt{2\pi + \alpha} = 2.50808$)

In[5909]:=

$$2\pi + \alpha /. \text{cRep}$$

Out[5909]=

$$6.29048$$

In[5910]:=

$$\sqrt{\%};$$

NRv1@%

Out[5911]//NumberForm=

$$2.50808346347244$$

JLN G_N value from eq. 451 using VEV (v)

In[5912]:=

(* JLN Prescription for G_N *)

$$(2\pi + \alpha) \hbar c \frac{\alpha^{16}}{v^2} /. \text{slRep}$$

% /. cRep

Out[5912]=

$$1.13692 c \alpha^{24} \hbar$$

MassUnit²

Out[5913]=

$$9.14221 \times 10^{-18} \text{ LengthUnit}^3$$

MassUnit TimeUnit²

```
In[5914]:=
GNtest = convert[%, SI]
Out[5914]=
6.67889 × 10-11 Meter3
Kilogram Second2
In[5915]:=
Clear@GNHopf;
simplify@NSolve[GNHopf == GNtest, GNHopf][[1, 1]]
Out[5916]=
GNHopf →  $\frac{6.67889 \times 10^{-11} \text{ Meter}^3}{\text{Kilogram Second}^2}$ 
In[5917]:=
GNHopf = 6.67480 × 10-11  $\frac{\text{Meter}^3}{\text{Kilogram Second}^2}$  /. slRep
Out[5917]=
6.6748 × 10-11 Meter3
Kilogram Second2
In[5918]:=
(*  $\frac{6.67430 \text{ Meter}^3}{\text{Kilogram Second}^2}$  **)
GravitationalConstant
GravitationalConstantError
Out[5918]=
6.674 × 10-11 Meter3
Kilogram Second2
Out[5919]=
1.5 × 10-15 Meter3
Kilogram Second2
In[5920]:=
GNSI
Out[5920]=
(6.67430 ± 0.00015) × 10-11 m3/ (kg s2)
In[5921]:=
GravitationalConstantError = 1.5`*^-15  $\frac{\text{Meter}^3}{\text{Kilogram Second}^2}$  /. slRep
Out[5921]=
1.5 × 10-15 Meter3
Kilogram Second2
JGM Symbolic Analysis:
In[5922]:=
(* JLN's GN ratio with most favorable exp. error *)
In[5923]:=
NRv1[gΔ1 =  $\frac{6.6748}{6.67445}$  /. slRep]
Out[5923]//NumberForm=
1.00005243877773
In[5924]:=
(* JLN's GN ratio with SI center value *)
NRv1[(**)*gΔ2 = (**)*simplify[ $\frac{\text{GNHopf}}{\text{GravitationalConstant}}$  /. slRep]]
Out[5924]//NumberForm=
1.00007491422322
JLN 's GN is outside the CURRENT std. error by 4 / 3 = 3.3333
the range quoted is a historically wide range 6.672 – 6.676 but is now 6.67430 (± 30)
In[5925]:=
doStdErr["GNHopf", "GravitationalConstant", 1,  $\frac{\text{Meter}^3}{\text{Kilogram Second}^2}$ ]
GNHopf =  $\frac{6.6748 \times 10^{-11} \text{ Meter}^3}{\text{Kilogram Second}^2}$ 
SI Std. Error =  $\frac{1.5 \times 10^{-15} \text{ Meter}^3}{\text{Kilogram Second}^2}$ 
New/Old Std. Error = 0.0000749142
Std. Error Factor (<1 is inside Experimental Error) = 3.33333
Higgs VEV (v) Constant:
JLN Supporting equations:
Higgs VEV (v) derived from Fermi Constant (GF) curated Wolfram MTM data and 2024 PDG CODATA
In[5926]:=
VEVSI
(* VEV (Wolfram curated is off by .2%, but v=1/√√2 GF is correct *)
VEVSI2 = convert[oldSI[ $\frac{\text{VEVSI}}{\text{cSI}^2}$  /. slRep], Giga eVperC2]
Out[5926]=
(3.9526 ± 0.0032) × 10-8 kg m2/s2
Out[5927]=
eVperC2 Giga (246.70 ± 0.20)
In[5928]:=
FermiCouplingConstantSI
Out[5928]=
(4.54379(57 ± 23) × 1014) s4/ (kg2m4)
In[5929]:=
(* GeV *)
FermiCouplingConstantPerGeV2
FermiConstantGeVPerC2 = oldSIInoUnc@FermiCouplingConstantPerGeV2
GFGeV2 = convert[FermiCouplingConstantPerkg2,  $\frac{1}{(\text{Giga eVperC2})^2}$ ]
Out[5929]=
4.54380 × 1014 Second4
Kilogram2 Meter4
Out[5930]=
4.54380 × 1014
Kilogram2
Out[5931]=
0.0000116638
eVperC22 Giga2
```

In[5932]:=

HiggsVacuumExpectationValue

HiggsVacuumExpectationValueError

Out[5932]=

$$3.95 \times 10^{-8} \text{ Kilogram Meter}^2$$

$$\text{Second}^2$$

Out[5933]=

$$3.20435 \times 10^{-11}$$

In[5934]:=

(* Convert G_F to VEV *)

$$\text{VEVSI3} = \text{convert}\left[1/\sqrt{\sqrt{2} \text{ GFGeV2}}, \text{Giga eVperC2}\right]$$

$$\text{VEVSI3Error} = \text{convert}\left[\text{HiggsVacuumExpectationValueError} \frac{\text{Kilogram Meter}^2}{\text{Second}^2}, \frac{\text{Giga eVperC2 Meter}^2}{\text{Second}^2}\right] / \left(\frac{\text{Giga eVperC2 Meter}^2}{\text{Second}^2}\right) /. \text{s1Rep}$$

Out[5934]=

246.22 eVperC2 Giga

Out[5935]=

$$1.79751 \times 10^{16}$$

Solving JLN G_N eq. 451 for VEV (v) :

In[5936]:=

Clear@v;

$$\text{simplify@NSolve}\left[\text{GravitationalConstant} == \text{convert}\left[\left(2 \pi + \alpha\right) \frac{\alpha^{16} \hbar c}{v^2}, \text{SI}\right], v\right][[2, 1]]$$

$$\text{vevHopf} = \text{convert}[\text{Values}\%, \text{Giga eVperC2}];$$

NRv1@%

$$\text{convert}[\%, \text{MassUnit}]$$

$$\% \alpha^4 /. \text{cRep};$$

NRv1@%

Out[5937]=

$$v \rightarrow 4.38945 \times 10^{-25} \text{ Kilogram}$$

Out[5939]//NumberForm=

246.23039926297080 eVperC2 Giga

Out[5940]=

$$8.29785 \times 10^8 \text{ MassUnit}$$

Out[5942]//NumberForm=

2.35302772639393 MassUnit

In[5943]:=

(* JLNs G_N ratio with most favorable exp. error *)

$$\text{Out}[-5] \text{ g}\Delta 1 /. \text{s1Rep};$$

NRv1@%

(* JLNs G_N ratio with the SI center value *)

$$\text{Out}[-7] \text{ g}\Delta 2 /. \text{s1Rep};$$

NRv1@%

Out[5944]//NumberForm=

246.24331128414730 eVperC2 Giga

Out[5946]//NumberForm=

246.24884542206340 eVperC2 Giga

In[5947]:=

$$v = \text{simplify}\left[\sqrt{\frac{2 \pi + \alpha}{\text{GN}}} \alpha^{16} \hbar c /. \text{cRep}\alpha\right]$$

$$\text{simplify}[\% /. \text{cRep}]$$

$$\text{simplify}[\% \alpha^4 /. \text{cRep}]$$

$$\text{vGeV} = \text{convert}[\%, \text{Giga eVperC2}]$$

Out[5947]=

$$0.937855 \text{ MassUnit} \frac{\sqrt{2 \pi + \alpha}}{\alpha^4}$$

Out[5948]=

$$8.295 \times 10^8 \text{ MassUnit}$$

Out[5949]=

2.35222 MassUnit

Out[5950]=

246.146 eVperC2 Giga

In[5951]:=

$$\text{vE} = \text{Convert}\left[v c^2 /. \text{cRep}, \text{Giga eVperC2} \left(\frac{\text{Meter}}{\text{Second}}\right)^2\right] /. \text{s1Rep}$$

Out[5951]=

$$2.21225 \times 10^{19} \text{ eVperC2 Giga Meter}^2$$

$$\text{Second}^2$$

JGM Symbolic Analysis:

In[5952]:=

(* G_N=1 *)

$$\text{NRv1@simplify}\left[\frac{2 \pi + \alpha}{\text{GN}} \frac{\alpha^{16} \hbar c}{v^2} /. \text{cRep}\alpha\right]$$

Out[5952]//NumberForm=

1.00000000000000

In[5953]:=

doStdErr["v", "VEVSI3", Giga eVperC2, Giga eVperC2, False, False]

v=246.146 eVperC2 Giga

SI Std. Error = 1.79751 × 10¹⁶

New/Old Std. Error = 0.000301189

Std. Error Factor (<1 is inside Experimental Error) = 4.12563 × 10⁻¹⁸


```
In[5954]:=
doStdErr["vevHopf", "VEVSI3", Giga eVperC2, Giga eVperC2, False, False]
Std. Error Factor (<1 is inside Experimental Error) = 5.83198×10-19
New/Old Std. Error = 0.000042576
SI Std. Error = 1.79751×1016
vevHopf=246.23 eVperC2 Giga
Std. Error Factor (<1 is inside Experimental Error) = 5.83198×10-19
New/Old Std. Error = 0.000042576
SI Std. Error = 1.79751×1016
vevHopf=246.23 eVperC2 Giga
Std. Error Factor (<1 is inside Experimental Error) = 5.83198×10-19
New/Old Std. Error = 0.000042576
SI Std. Error = 1.79751×1016
vevHopf=246.23 eVperC2 Giga
Std. Error Factor (<1 is inside Experimental Error) = 5.83198×10-19
New/Old Std. Error = 0.000042576
SI Std. Error = 1.79751×1016
vevHopf=246.23 eVperC2 Giga
Std. Error Factor (<1 is inside Experimental Error) = 5.83198×10-19
New/Old Std. Error = 0.000042576
SI Std. Error = 1.79751×1016
vevHopf=246.23 eVperC2 Giga
Std. Error Factor (<1 is inside Experimental Error) = 5.83198×10-19
New/Old Std. Error = 0.000042576
SI Std. Error = 1.79751×1016
vevHopf=246.23 eVperC2 Giga
```

Hubble Parameter (H_0 ($\simeq 68$) CMB|($\simeq 73$) Sn **in tension**), Cosmological Constant (Λ) and $\Omega_{\text{Total}} = \sum \{\text{Dark Energy } (\Omega_{\Lambda=\text{DE}}), \text{Dark Matter } (\Omega_{\text{DM}}), \text{visible Matter } (\Omega_{\text{m}})\}$:

JLN reference snapshots:

```
In[5955]:=
Numerical prediction. Using (485) and the spectral value of  $\alpha$  (Theorem 106),

$$\Lambda = 3 \exp\left(-\frac{2 + \zeta(3)/24}{\alpha}\right) = 2.94 \times 10^{-122} \quad (\text{Planck units}),$$

against the observed  $\Lambda_{\text{obs}} = 2.85 \times 10^{-122}$ . The dark-energy density is constrained observationally to about 1.9% (dominated by the  $H_0$  uncertainty through  $\Lambda \propto \Omega_{\Lambda} H_0^2$ ), so the prediction lies  $1.7\sigma$  above the measured value with no free parameters. Through the Friedmann relation  $\Lambda = 3\Omega_{\Lambda} H_0^2/c^2$ , this corresponds to
```

$$H_0 = 68.5 \text{ km s}^{-1} \text{ Mpc}^{-1}, \tag{486}$$

which lies 2σ above the *Planck* CMB value (67.4 ± 0.5) and 4.4σ below the local distance-ladder value (73.0 ± 1.0). Equation (486) is therefore a Planck-side prediction, parameter-free and independent of which side of the Hubble tension is correct: if converging measurements settle near 68.5, the prediction is supported; if they settle near 73.0, it is falsified. The 1.7σ offset is a real, open residual; no correction term is posited.

```
In[5956]:=
Numerical prediction. Using (485) and the spectral value of  $\alpha$  (Theorem 106),

$$\Lambda = 3 \exp\left(-\frac{2 + \zeta(3)/24}{\alpha}\right) = 2.94 \times 10^{-122} \quad (\text{Planck units}),$$

against the observed  $\Lambda_{\text{obs}} = 2.85 \times 10^{-122}$ . The dark-energy density is constrained observationally to about 1.9% (dominated by the  $H_0$  uncertainty through  $\Lambda \propto \Omega_{\Lambda} H_0^2$ ), so the prediction lies  $1.7\sigma$  above the measured value with no free parameters. Through the Friedmann relation  $\Lambda = 3\Omega_{\Lambda} H_0^2/c^2$ , this corresponds to
```

$$H_0 = 68.5 \text{ km s}^{-1} \text{ Mpc}^{-1}, \tag{486}$$

which lies 2σ above the *Planck* CMB value (67.4 ± 0.5) and 4.4σ below the local distance-ladder value (73.0 ± 1.0). Equation (486) is therefore a Planck-side prediction, parameter-free and independent of which side of the Hubble tension is correct: if converging measurements settle near 68.5, the prediction is supported; if they settle near 73.0, it is falsified. The 1.7σ offset is a real, open residual; no correction term is posited.

JLN Supporting equations:

```
In[5957]:=
(* If the tension is not "a real thing", then Wolfram (and I) prefer to use CMB vs. SuperNova (Sn) distance ladder *)
```

```
In[5958]:=
HubbleConstant
NRv1@convert[%,  $\frac{\text{Kilo Meter}}{\text{Second Mega Parsec}}$ ]
Out[5958]=
2.2×10-18
Second
Out[5959]//NumberForm=
67.66049157316472 Kilo Meter
Mega Parsec Second
In[5960]:=
(* c H0 *)
In[5961]:=
NRv1[cH0 = %% oldSI@cSI /. slRep]
Out[5961]//NumberForm=
(2.02841050782073×1010) Kilo Meter2
Mega Parsec Second2
```

```
e.q. 486
In[5962]:=
(* JLN  $\Delta_{\text{obs}}$  is given as  $2.85 \cdot 10^{-122}$  *)
In[5963]:=
 $\Delta_{\text{obs}} = \text{CosmologicalConstant}(\text{lp Meter})^2 /. \text{slRep}$ 

$$\Delta_{\text{th}} = 3 \text{ e}^{-\frac{2+\zeta(3)/24}{\alpha}};$$

NRv1[% /. cRep]
Out[5963]=
2.88794×10-122
Out[5965]//NumberForm=
2.94012102576161×10-122
In[5966]:=

$$\Delta_{\text{per}\Omega} = 3 \times \left(\frac{\text{HubbleConstant}}{\text{SpeedOfLight}}\right)^2;$$

Clear@ $\Delta_{\text{per}\Omega} \mathbf{x}$ ;
NRv1[ $\Delta_{\text{per}\Omega} \mathbf{x} = \Delta_{\text{per}\Omega} /. \mathbf{x} \rightarrow 1 /. \text{slRep}$ ]
Out[5968]//NumberForm=
1.60488066668929×10-52
Meter2
In[5969]:=
(* Solve for equivalence to the observed CosmologicalConstant *)
In[5970]:=
NRv1[ $\Delta_{\text{per}\Omega} \Delta_{\text{sol}} = \text{Solve}[\Delta_{\text{per}\Omega} / \text{CosmologicalConstant} == 1 /. \text{slRep}, \mathbf{x}] \llbracket 1 \rrbracket$ ]
Out[5970]//NumberForm=
{ $\mathbf{x} \rightarrow 0.68889981850223$ }
In[5971]:=
(* Dimensionless Planck scaled  $\Delta_{\text{per}\Omega}$  *)
```

```
In[5972]:=
AperΩΔDimless = AperΩΔ (1P Meter)^2 /. s1Rep;

NRv1[% /. x → 1]
Out[5973]//NumberForm=
4.19211096756799×10-122

JGM Symbolic Analysis:
In[5974]:=
  Δth
----- /. cRep
  Δobs
Out[5974]=
1.01807

In[5975]:=
(* ΩΔ @ ≈70% of ΩTot *)
ΩΔth = Δth / AperΩΔDimless /. Join[{x → 1}, cRep, s1Rep]
Δobs / AperΩΔDimless /. Join[{x → 1}, cRep, s1Rep]
(* But this is just the dimensionless h=H0/100 km/(s-Mpc) *)
Δobs / AperΩΔDimless /. Join[AperΩΔsol, cRep, s1Rep]
Out[5975]=
0.701346
Out[5976]=
0.6889
Out[5977]=
1.
```

Show that cH₀ is the "AccelerationAssociatedWithCosmologicalExpansionRate" :

```
In[5978]:=
doStdErr["cH0", "AccelerationAssociatedWithCosmologicalExpansionRate",
  Meter/Second^2, Meter/Second^2]
cH0=
  6.57358×10-10 Meter
  -----
  Second^2
SI Std. Error = 5.24643×10-12
New/Old Std. Error = 0.00445368
Std. Error Factor (<1 is inside Experimental Error) = 0.555556
```

The JLN value of ΔperΩΔx = $\frac{\Delta_{obs}}{\Delta_{th}}$ is outside the CURRENT std. error by a **factor of ≈ 30**.

```
In[5979]:=
doStdErr["AperΩΔx", "CosmologicalConstant", Meter^-2, Meter^-2]
  1.6×10-52
AperΩΔx=
  Meter^2
  -----
  1.6×10-54
SI Std. Error = 1.6×10-54
New/Old Std. Error = 0.5
Std. Error Factor (<1 is inside Experimental Error) = 31.2049
```

Unless adding the h scaling factor :

```
In[5980]:=
(* Adding the solution factor of 0.688... for equivalence *)
```

```
NRv1[AperΩΔx = AperΩΔ /. ΔperΩΔsol]
doStdErr["AperΩΔx", "CosmologicalConstant", Meter^-2, Meter^-2]
Out[5980]//NumberForm=
1.10560200000000×10-52
AperΩΔx=
  Meter^2
  -----
  1.×10-52
```

```
SI Std. Error = 1.6×10-54
New/Old Std. Error = 0.×10-1
Std. Error Factor (<1 is inside Experimental Error) = 0.
```

Physical Constants (α, ħ, c)

Fine Structure Constant (α):

JLN reference snapshots:

7.3 Electric Charge and the Fine-Structure Constant

Theorem 105 (Electric Charge Quantization). (Derived.) $q = e \cdot c_1$, $c_1 \in \mathbb{Z}$.

Proof. The integrality condition $\frac{1}{2\pi} \int_{\mathbb{CP}^1} F = n \in \mathbb{Z}$ (cocycle condition on $\mathcal{L} \rightarrow \mathbb{CP}^1$) quantizes charge in integer multiples of e . □

Theorem 106 (Fine-Structure Constant). (Derived, with one physical identification marked below.)

$$\alpha = \frac{2 \operatorname{Vol}(S^2)}{\operatorname{Vol}(S^4)^2 \cdot \operatorname{Vol}(\mathbb{RP}^1)} \cdot \left[\frac{\operatorname{Vol}(S^9)}{2^5 \cdot 5} \right]^{1/4} = \frac{1}{137.0360824 \dots}$$

Experiment: $\alpha_{\text{exp}}^{-1} = 137.0359991(2)$; *agreement to six significant figures* (0.00006%).

Proof. The fine-structure constant is the coupling of the $U(1)$ fiber to the total space—the standard gauge-theory definition of a coupling constant, here computed geometrically rather than measured. The universal bundle is $S^1 \rightarrow S^\infty \rightarrow \mathbb{CP}^\infty$; its finite approximation containing all Standard Model sectors is $S^1 \rightarrow S^9 \rightarrow \mathbb{CP}^4$, which we use throughout.

Step 1 (*derived*). The O’Neill A -tensor [86] on $S^1 \rightarrow S^9 \rightarrow \mathbb{CP}^4$ gives fiber curvature fraction $f(4) = 1/(2 \cdot 4 + 1) = 1/9$. Photon transverse degrees of freedom on S^2 (emission + absorption) give fiber spectral weight $\mathcal{W}_{\text{fiber}} = 2 \operatorname{Vol}(S^2) = 8\pi$.

Step 2 (*derived*). The total gauge spectral weight is $\mathcal{W}_{\text{total}} = \operatorname{Vol}(S^4)^2 \cdot \operatorname{Vol}(\mathbb{RP}^1) = 64\pi^5/9$: two copies of $\operatorname{Vol}(S^4)$ for the squared amplitude e^2 and $\operatorname{Vol}(\mathbb{RP}^1) = \pi$ for the projective identification of the real gauge field.

Step 3 (*derived*). The partition function $Z \propto (\det\{\}^t \mathcal{B})^{-1/2} = (\det\{\}^t \Delta)^{-1/4}$ determines the normalization. The Hua volume [87] $V(D_n) = \pi^n/n!$ of the unit ball $D_n \subset \mathbb{C}^n$ gives spectral volume $V_{\text{spec}}(S^{2n-1}) = \operatorname{Vol}(S^{2n-1})/(2^n \cdot n)$ via the Bergman kernel boundary relation $\operatorname{Vol}(S^{2n-1})/V(D_n) = 2n$ and the 2^{n-1} orientational factor from the Hopf $U(1)$ fibration. At $n = 5$: $\mathcal{N}_{\mathcal{B}} = [\operatorname{Vol}(S^9)/160]^{1/4}$.

Step 4 (*uniqueness of the electromagnetic coupling*). We show that the electromagnetic coupling is the *unique* dimensionless invariant of the $U(1)$ fiber sector on S^9 satisfying four necessary conditions. □

JLN Supporting equations:

```
In[5983]:=
Wfiber = 2 Vol(S^2) = 8π.;
vs2 = 8 π;

; Wtotal = Vol(S^4)^2 . Vol(RP^1) = 64π^5/9;
In[5986]:=
vs4vrp = 64 π^5 / 9;
```

```
In[5987]:=
The Hua volume [61]  $V(D_n) = \pi^n/n!$  of the unit ball  $D_n \subset \mathbb{C}^n$ ;
In[5988]:=
vdn :=  $\frac{\pi^\#}{\#!}$  &;
In[5989]:=
[1]  $\text{Vol}(S^{2n-1})/V(D_n) = 2n$  ;
In[5990]:=
vs9 = 2 × 5 vdn@5
Out[5990]=
 $\pi^5$ 
12
JGM Symbolic Analysis:
JLN  $\alpha$  value from Wyler’ s 1969 constantfactor is "accurate to 6 figures and 0.00006%",
but it is still outside the CURRENT std. error by a wide margin (factor of  $\approx 4000$ )
In[5991]:=
NRvl[ $\alpha$ Hopf =  $\frac{\text{vs2}}{\text{vs4vrp}} \left( \frac{\text{vs9}}{5 \times 2^5} \right)^{1/4} /. \text{s1Rep}$ ]

NRvl@ $\frac{1}{\%}$ 
Out[5991]//NumberForm=
0.00729734813003
Out[5992]//NumberForm=
137.03608244816430
In[5993]:=
NRvl@FineStructureConstant
NRvl[1 / % /. s1Rep]
Out[5993]//NumberForm=
0.00729735256430
Out[5994]//NumberForm=
137.03599917759010
In[5995]:=
FineStructureConstantError
Out[5995]=
 $1.16758 \times 10^{-12}$ 
In[5996]:=
NRvl[ $\alpha\Delta 1 = \frac{\alpha}{\alpha\text{Hopf}} /. \text{cRep}$ ]
Out[5996]//NumberForm=
1.00000060765474
In[5997]:=
doStdErr[" $\alpha$ Hopf", "FineStructureConstant", 1, 1, False, False]

$$\alpha\text{Hopf} = \frac{16 \times 2^{3/4} \times 5^{1/4} \pi^{11/4}}{3 \times 3^{3/4}}$$

SI Std. Error =  $1.16758 \times 10^{-12}$ 
New/Old Std. Error =  $6.08 \times 10^{-7}$ 
Std. Error Factor (<1 is inside Experimental Error) = 3797.84
```

Planck Constant ($\hbar$):

JLN reference snapshots:

In[5998]:=

Corollary 24 (Angular Momentum Quantization). *The winding number $w \in \pi_1(S^1) \cong \mathbb{Z}$ of the horizontal lift of a closed loop in $\mathbb{CP}^4$ gives the eigenvalue of $\hat{L}_z = -i\hbar\partial_\phi$; periodicity of $e^{iw\phi}$ forces $L_z = \hbar w$ with $w \in \mathbb{Z}$.*

Definition 16 (Planck’s Constant|from the Fiber Cross-Section). *The S^1 fiber is the minimal-dimensional sub-manifold of the Hopf hierarchy. Its cross-sectional diameter—the physical width of the fiber considered as a tube—is the Planck length:*

$$\ell_P := \text{cross-sectional diameter of the } S^1 \text{ fiber.}$$

Since the speed of light c (Theorem [103]) and Newton’s constant G (Theorem [108]) are both derived, Planck’s reduced constant follows:

$$\hbar = \frac{\ell_P^2 c^3}{G} = 1.054571817 \times 10^{-34} \text{ J} \cdot \text{s}, \qquad h = 2\pi\hbar.$$

Status. The identification of the fiber cross-section with ℓ_P is a definition: the S^1 fiber is the smallest geometric object in the Hopf hierarchy, and the Planck length is the smallest physical length. Given this identification, $\hbar$ is determined by two already-derived constants (c and G) and is not an independent input. Quantization arises because the fiber has a finite cross-section: only discrete modes fit inside a tube of finite cross-section.

JLN Supporting equations:

```
In[5999]:=
ħ // . cRep
SI@%
NATURAL@%
Out[5999]=
1.24359 × 1017 LengthUnit2 MassUnit
TimeUnit
Out[6000]=
1.05457 × 10-34 Kilogram Meter2
Second
Out[6001]=
1. NaturalLengthUnit2 NaturalMassUnit
NaturalTimeUnit
```

SpeedOfLight (c):

JLN reference snapshots:

Derivation of the Speed of Light from Topology in TUFT

Within TUFT, the speed of light c arises from the intrinsic ratio of topological volumes and the writhe $\mathcal{W}$ of the S^1 fiber in the principal bundle

$$S^1 \rightarrow S^9 \overset{\pi}{\rightarrow} \mathbb{CP}^4,$$

where $\mathbb{CP}^4$ carries spatial structure.

Normalize the Kähler form ω on $\mathbb{CP}^4$ such that

$$\int_{\mathbb{CP}^1} \omega = 2\pi,$$

yielding the total symplectic volume

$$\text{Vol}(\mathbb{CP}^4) = \frac{(2\pi)^4}{24} \approx 64.9394.$$

The volume of the unit 8-sphere is

$$\text{Vol}(S^8) = \frac{2\pi^{9/2}}{\Gamma(9/2)} \approx 29.655.$$

Define the scale ratio

$$R_s := \left(\frac{\text{Vol}(\mathbb{CP}^4)}{\text{Vol}(S^8)}\right)^{1/8} \approx 1.103,$$

and the writhe of the fiber knot,

$$\mathcal{W} = \left(\frac{35}{16}\right)^{1/8} \approx 1.1027.$$

Topological consistency requires

$$R_s = \mathcal{W},$$

yielding the dimensionless speed of light

$$c_{\text{dimless}} = \frac{R_s}{\mathcal{W}} = 1.$$

102

JLN Supporting equations:
CP4vol/S8vol Rs & W (from June 2025 version)

```
In[6003]:=
CP4vol =  $\frac{(2 \pi)^4}{4 !}$ 

NRv1@%
Out[6003]=
 $2 \pi^4$ 

64.93939402266828
Out[6004]//NumberForm=
64.93939402266828

In[6005]:=
Γ@ $\frac{9}{2}$ 

NRv1@%
Out[6005]=
 $105 \sqrt{\pi}$ 

11.63172839656745
Out[6006]//NumberForm=
11.63172839656745

In[6007]:=
S8vol =  $\frac{2 \pi^{\frac{9}{2}}}{\Gamma@ \frac{9}{2}}$ 

NRv1@%
Out[6007]=
 $32 \pi^4$ 

29.68658012464836
Out[6008]//NumberForm=
29.68658012464836

In[6009]:=
 $\frac{16}{24} \bigg/ \frac{32}{105}$ 

NRv1@%
Out[6009]=
 $35$ 

2.18750000000000
Out[6010]//NumberForm=
2.18750000000000

In[6011]:=
 $\frac{35}{16}$ 

7 × 5
Out[6011]=
 $2^4$ 

35
Out[6012]=
 $35$ 

In[6013]:=
CP4vol + S8vol

NRv1@%
Out[6013]=
 $34 \pi^4$ 

94.62597414731670
Out[6014]//NumberForm=
94.62597414731670
```



```
In[6015]:=
CP4vol
S8vol
Out[6015]=
35
16
JGM Symbolic Analysis:
In[6016]:=
Rs = (CP4vol/S8vol)^(1/8)

W = (35/16)^(1/8)

NRv1@%
%%% == %%
Out[6016]=
35^(1/8)
Out[6017]=
35^(1/8)
Out[6018]//NumberForm=
1.10279174804171
Out[6019]=
True
```

unit length/time (L_{topo}/T_{topo}):

L_{topo} $\frac{\hbar}{c}$ (ML) $\rightarrow \frac{\hbar}{c} \frac{1}{v=M}$ =L vs. $\hbar c$ (Mc²L) $\rightarrow \hbar c \frac{1}{v=M}$ = c²L (from June 21, 2026 - omitted in July version)

JLN reference snapshots:

In[6020]:=

5.2. The Geometric Unit System

Theorem 30 (The Speed of Light). (Derived.) *Let $S^{2n+1}(R)$ carry the round metric decomposed via the canonical connection as $g = R^2 d\phi^2 + g_H$. Define the causal metric $g_{\text{causal}} = -R^2 d\phi^2 + g_H$. Then null geodesics propagate at*

$$c_{\text{geom}} = 1$$

in geometric units (fiber-lengths per fiber-time).

Proof. A null curve satisfies $-R^2 \dot{\phi}^2 + g_H(\dot{x}, \dot{x}) = 0$, giving $\sqrt{g_H(\dot{x}, \dot{x})}/(R|\dot{\phi}|) = 1$. The canonical connection assigns the same curvature radius R to the fiber and horizontal slices, so the ratio is unity identically. $\square$

Theorem 31 (Holonomy Quantization). (Derived—purely topological.) *Let $A = \alpha d\phi$ be a $U(1)$ connection on the S^1 fiber. The line bundle $\mathcal{L} = S^3 \times_{U(1)} \mathbb{C}$ admits well-defined sections only if $\alpha \in \mathbb{Z}$. The minimal nontrivial holonomy is*

$$S_{\min} = \oint_{\gamma} A = 2\pi$$

in geometric units, corresponding to one complete phase rotation $e^{2\pi i}$.

Proof. The cocycle condition $g_{ij}g_{jk}g_{ki} = 1$ on triple overlaps requires $e^{2\pi i\alpha} = 1$, hence $\alpha \in \mathbb{Z}$. $\square$

Theorem 32 (Physical Width of the S^1 Fiber). (Derived from Axiom 1 and Theorem 31) *The physical circumference of the S^1 fiber is*

$$L_{\text{topo}} = \frac{1}{v} \text{ (natural units),} \quad L_{\text{topo}} = \frac{\hbar c}{v} \approx 8.014 \times 10^{-19} \text{ m (SI).}$$

Proof. By Axiom 1, the physical connection strength is $|A| = v$ (natural units). The holonomy around one fiber circuit is $\oint A = v \cdot L$. By Theorem 31, the minimal nontrivial holonomy is 2π , which in natural units ($\hbar = 1$) corresponds to one action quantum. Setting $v \cdot L = 1$ gives $L = 1/v$. Restoring SI: $L_{\text{topo}} = \hbar c/v$. $\square$

Definition 8 (Planck’s Constant). (Definition—forced by dimensional uniqueness.) *The physical action of one complete S^1 fiber circuit (holonomy $S_{\min} = 2\pi$ geometric units) defines Planck’s constant h and its reduced form $\hbar$:*

$$h := \text{SI action of one fiber circuit} = 6.62607015 \times 10^{-34} \text{ J} \cdot \text{s}, \quad \hbar = \frac{h}{2\pi}.$$

The factor 2π is the geometric holonomy of one circuit (Theorem 31); h is the physical action of that circuit; $\hbar = h/(2\pi)$ is the action per radian.

Status. Holonomy quantization is a topological theorem. The identification of one fiber circuit with h is a definition: it bridges geometric and SI units. It is forced by dimensional uniqueness— h is the unique physical constant with dimensions of action, and the fiber holonomy is the unique topologically quantized action in the fibration—but it is not derived from the action functional. Given this definition and v , the SI values of c and L_{topo} follow from $c = L_{\text{topo}}/T_{\text{topo}}$ and $L_{\text{topo}} = \hbar c/v$.

JLN Supporting equations:
after eq. 3 - L_{topo} Section 3.3 (and 5.2+ after eq, 137) again

```
In[6021]:=
Ltopo = simplify@h/(c v) /. s1Rep

% /. cRep
% alpha^-4 /. cRep
1 / %
convert[Ltopo, SI]
Out[6021]=
1.06626 alpha^4 h
c MassUnit sqrt(2) pi + alpha
Out[6022]=
1.20555 x 10^-9 LengthUnit
Out[6023]=
0.425131 LengthUnit
Out[6024]=
2.35222
LengthUnit
Out[6025]=
8.01668 x 10^-19 Meter
```

In[6026]:=

Ttopo = simplify@ $\frac{\text{Ltopo}}{c}$ /. s1Rep

% /. cRep

convert[%%, Second]

Out[6026]=

$1.06626 \alpha^4 \hbar$

$c^2 \text{MassUnit} \sqrt{2 \pi + \alpha}$

Out[6027]=

$9.69407 \times 10^{-27} \text{TimeUnit}$

Out[6028]=

$2.67408 \times 10^{-27} \text{Second}$

In[6029]:=

% α^{-12} /. cRep

Out[6029]=

0.117271 Second

Particle ($\simeq$ bare rest) Masses (that run)

JLN reference snapshots:

In[6030]:=

(v) *Evaluation of $\det\{\}'B_n$ via the Sector Determinant Lemma yields the universal mass formula*

$$m_n = \Lambda_{\text{shell}} (n+1) \exp(a n - \zeta(3) n^2) \phi_n, \quad n = 1, 2, 3,$$

where Λ_{shell} is the shell-specific dimensional scale set by the Fermi constant, $(n+1)$ is the $SU(2)$ multiplicity, a is the helicity coefficient, $\zeta(3) n^2$ is the Casimir determinant suppression, and ϕ_n is the knot-complement spectral correction. Each coefficient is derived, not fit:

Coefficient	Geometric origin	Derived in
Λ_{shell}	$\sqrt{2\pi} v \kappa^6$; unit conversion $\times$ spectral coupling	Axiom 3 , eq. (346)
$(n+1)$	dim of $SU(2)$ representation at winding n	Peter-Weyl
$a = \kappa \cdot \gamma_{\text{eff}} \cdot \ell$	Hopf self-linking ($\ell=6$) $\times$ Clifford radius $\times$ spectral determinant	Thm 93
$\zeta(3) n^2$	Torsion exponent $\sigma_3 = \zeta(3)/(4\pi^2)$ from Beltrami zeta on S^3 ; lens space determinant	Lemma 13
ϕ_n	Zeta-regularized determinant ratio $\det_\zeta B_\Theta / \det_\zeta B$ on $S^3 \setminus T(2, n)$	eq. (321)

;

--

No coefficient is a free parameter. The helicity coefficient a is derived in [§6.13](#) from three geometrically forced inputs (the Chern-Simons coupling κ , the Clifford helicity scale γ_{eff} , and the framing number $\ell = 6$ from the trefoil's Hopf self-linking). The torsion exponent $\zeta(3)$ is proved via the Sector Determinant Lemma using Nash-O'Connor lens space determinants. The correction ϕ_n is the Atiyah-Patodi-Singer spectral invariant of the knot complement $S^3 \setminus T(2, n)$, computable from the Seifert geometry of each torus knot complement.

In[6031]:=

6.3 Shell Specialization

The nested Hopf geometry stratifies the Beltrami spectrum into distinct topological shells, each hosting a different class of particle modes:

Shell	Geometry	Particles	Mechanism
S^1	$U(1)$ fiber	Photon, graviton	Connection (unknot) and its torsion (figure-eight)
S^3	$\cong SU(2)$	Leptons, $W^\pm$, Z , H	$SU(2)$ eigenmodes; masses from coexact 1-form knots
S^5	$\cong SU(3)/SU(2)$	Quarks	Color triplets; masses from coexact 2-forms
S^7	$\cong SU(4)/SU(3)$	Gluons	$SU(3)$ gauge connection; massless ($\lambda_{\min} = 0$); confined
S^9	$\cong SU(5)/SU(4)$	Neutrinos	Color singlets; PMNS mixing from $H^*(\mathbb{CP}^4)$; mass-suppressed

The known physical particle content of the Standard Model is exhausted by S^1 , S^3 , S^5 , S^7 , and S^9 .

6.4 The Beltrami Operator on the Hopf Shell

We construct, from first principles, the spectral dynamics governing the torsion sector on the Hopf shell

$$S^1 \longrightarrow S^3 \longrightarrow \mathbb{CP}^1.$$

The construction begins with the torsion functional, reduces canonically to a quadratic form on 1-forms, and leads naturally to the first-order Beltrami operator whose spectrum controls the dynamics.

Comparison with the S^3 and S^9 helicity coefficients. The four shells differ because each contributes its own Clifford radius, contact normalization, spectral zeta, and framing mechanism:

	S^3 (leptons)	S^5 (quarks)	S^7 (gluons)	S^9 (neutrinos)
Clifford radius r_{Cliff}	$1/\sqrt{2}$	$1/\sqrt{3}$	$1/\sqrt{4}$	$1/\sqrt{5}$
Contact norm. $\mathcal{N}$	$4\pi^2$	$8\pi^3$	$16\pi^4$	$32\pi^5$
Framing ℓ	6 (knot)	6 (knot)	56 ($\Lambda^3(SO(8))$)	16 (contact chirality)
Spectral input	$\zeta_{\mathcal{B}}'(0)$	spectral ₅	$\zeta_{\mathcal{B}_7}'(0)$	$\zeta_{\Delta_2}'(0)$
Universal phase order	α^1	α^2	α^3	α^4
Result	$a \approx 8.528$	$a_5 \approx 3.564$	(massless)	$a_9 \approx \sqrt{5}$

The coefficient a_5 is universal across the three quark winding sectors because ℓ is a global framing invariant of the Hopf-framed Beltrami domain (fixed by the maximal orbit $T(2,3)$), not a property of any individual sector's knot type. Sector dependence enters through the winding number n multiplying a_5 , through the quadratic Ray–Singer term $C_5 n^2$, and through the sector correction $\delta_n^{(5)}$.

Remark 52 (Normalization choices are geometrically forced). *Three normalizations enter the derivation of a_5 . None is a free parameter.*

- The framing number $\ell = 6$.** This is the Hopf self-linking number $\text{sl}_{\text{Hopf}}(T(2,3)) = 2 \cdot 3 = 6$ of the trefoil, which is the maximal generational orbit. The trefoil is the last entry in the generational ladder before the integrable-to-hyperbolic transition at $k = 4$ forces the Beltrami flow off the Clifford torus foliation. Thus $\ell = 6$ is fixed by the three-generation corollary, not chosen.
- The Clifford radius $r_{\text{Cliff}}^{(5)} = 1/\sqrt{3}$.** All three generational orbits $T(2,n)$ lie on the Clifford torus $T_{\text{Cliff}}^2 \subset S^5$ by the Minimal-Level Integrable Rigidity theorem. The Clifford torus in $\mathbb{C}^3$ has all three coordinates equal: $|z_j| = 1/\sqrt{3}$ for $j = 1, 2, 3$. Normalizing the helicity flux by the radius of the surface on which the orbits live is the unique geometrically consistent choice.
- The Chern–Simons level $k = \ell = 6$.** As on S^3 , the Chern–Simons theory on the Beltrami domain is defined with respect to the Hopf framing. Consistency between the framing and the quantization requires $k = \ell$. Any other identification would produce a mismatch between the topological charge quantization of the CS theory and the geometric framing of the domain on which it is defined.

Theorem 96 (Uniqueness of the S^5 helicity coefficient). *Let a_5 be a real constant satisfying:*

- a_5 is the linear-in- n coefficient of $\ln \det' \mathcal{B}_{5,n}$ for the Beltrami operator on S^5 restricted to the n th fiber winding sector;
- a_5 is constructed solely from intrinsic spectral and geometric invariants of the Hopf-framed Beltrami domain on the unit round S^5 ;
- a_5 is consistent with the Chern–Simons quantization condition and the framing determined by the maximal integrable orbit.

Then

$$a_5 = \exp\left(\frac{\text{spectral}_5}{6}\right) \sqrt{3} \left(2 + \frac{\zeta(3)}{4\pi^2}\right).$$
 (406)

Proof. The proof follows the same three-step structure as the S^3 uniqueness theorem (Theorem 93).

Step 1: The framing number $\ell = 6$ is unique. By the Three-Generation Theorem 87, the maximal integrable Beltrami level is $k = 3$, corresponding to the trefoil $T(2,3)$. Its Hopf self-linking is $\text{sl}_{\text{Hopf}}(T(2,3)) = 2 \cdot 3 = 6$. The Chern–Simons level on a framed manifold equals the self-linking number of the maximal orbit. Therefore $\ell = 6$ is the unique value compatible with conditions (i) and (iii).

Step 2: The Clifford scale $\gamma_{\text{eff}}^{(5)} = 8\pi^3\sqrt{3}$ is unique. All three generational orbits $T(2,n)$ lie on the Clifford torus $T_{\text{Cliff}}^2 \subset S^5$ by the Minimal-Level Integrable Rigidity theorem. The helicity functional $\mathcal{H}[C_n] = \int C_n \wedge dC_n$ evaluated on orbits confined to T_{Cliff}^2 factors as $\mathcal{H} = (\text{orbital integral}) \times (\text{transverse scale})$. The transverse scale is

JLN Supporting equations:

Section 6 Particle Mass Spectrum from Eigenvalues of the Beltrami–Hodge–Star Flow on the Universal Action

eq. 336-343 6.12 Unified Knot-Complement Spectral Coupling

eq. 336 Confirmation via the Cheeger–Müller theorem

In[6033]:=

$\mathcal{B} := \text{1n} - \text{n}^2 \, \zeta@3 + \text{01} /. \text{slRep} \, \&;$

$\mathcal{B}@3$

Out[6034]=

$\text{1n} + \text{01} - \text{64 Zeta}[3]$

eq. 339-340 dPD Poincaré duality factor (Section 6.12)

```
In[6035]:=
dPD[p_, k_] := Which[
  p == 1 && k == 1, 1,
  p == 2 && k == 2, 4,
  p == 2 && k == 4, 2,
  True, 1];
Table[{ {p, k}, dPD[p, k], dPD[p, k] },
  {p, 2}, {k, 4}] // MatrixForm
```

Out[6036]//MatrixForm=

$$\left(\begin{array}{c} \left(\begin{array}{c} \{1, 1\} \\ 1 \\ 1 \end{array}\right) \left(\begin{array}{c} \{1, 2\} \\ 1 \\ 1 \end{array}\right) \left(\begin{array}{c} \{1, 3\} \\ 1 \\ 1 \end{array}\right) \left(\begin{array}{c} \{1, 4\} \\ 1 \\ 1 \end{array}\right) \\ \left(\begin{array}{c} \{2, 1\} \\ 1 \\ 1 \end{array}\right) \left(\begin{array}{c} \{2, 2\} \\ 4 \\ 4 \end{array}\right) \left(\begin{array}{c} \{2, 3\} \\ 1 \\ 1 \end{array}\right) \left(\begin{array}{c} \{2, 4\} \\ 2 \\ 2 \end{array}\right) \end{array}\right)$$

Unified Knot-Complement Spectral Coupling

In[6037]:=

$$\sigma 3 = \frac{\zeta @3}{(2 \pi)^2};$$

In[6038]:=

$$\sigma k[p_, k_] := \sigma 3 \frac{1}{dPD[p, k]} /. s1Rep;$$

```
Table[{ {p, k}, \sigma k[p, 2^k], NRV1@\sigma k[p, 2^k] },
  {p, 2}, {k, 0, 2}] // MatrixForm
```

Out[6039]//MatrixForm=

$$\left(\begin{array}{c} \left(\begin{array}{c} \{1, 0\} \\ \frac{Zeta[3]}{4 \pi ^2} \\ 0.03044845705839 \end{array}\right) \left(\begin{array}{c} \{1, 1\} \\ \frac{Zeta[3]}{4 \pi ^2} \\ 0.03044845705839 \end{array}\right) \left(\begin{array}{c} \{1, 2\} \\ \frac{Zeta[3]}{4 \pi ^2} \\ 0.03044845705839 \end{array}\right) \\ \left(\begin{array}{c} \{2, 0\} \\ \frac{Zeta[3]}{4 \pi ^2} \\ 0.03044845705839 \end{array}\right) \left(\begin{array}{c} \{2, 1\} \\ \frac{Zeta[3]}{16 \pi ^2} \\ 0.00761211426460 \end{array}\right) \left(\begin{array}{c} \{2, 2\} \\ \frac{Zeta[3]}{8 \pi ^2} \\ 0.01522422852920 \end{array}\right) \end{array}\right)$$

eq. 344-355 Section 6.13 Helicity Flux a from Hopf Self-Linking, Clifford Geometry and the Beltrami Determinant
Beltrami Determinant with Apéry’s constant $\zeta @3$ and S^9

```
In[6040]:=
{#, \zeta @#, NRV1@\zeta @#} & /@ (2 Range@8 - 1) // MatrixForm
```

Out[6040]//MatrixForm=

$$\left(\begin{array}{c} 1 \text{ ComplexInfinity } \text{ComplexInfinity} \\ 3 \text{ Zeta}[3] \text{ 1.20205690315959 } \\ 5 \text{ Zeta}[5] \text{ 1.03692775514337 } \\ 7 \text{ Zeta}[7] \text{ 1.00834927738192 } \\ 9 \text{ Zeta}[9] \text{ 1.00200839282608 } \\ 11 \text{ Zeta}[11] \text{ 1.00049418860412 } \\ 13 \text{ Zeta}[13] \text{ 1.00012271334758 } \\ 15 \text{ Zeta}[15] \text{ 1.00003058823631 } \end{array}\right)$$

eq. 345 Clifford geometric normalization & Effective Chern–Simons coupling from the Beltrami determinant

In[6041]:=

$$rCliff = 1/\sqrt{2};$$
$$\gamma eff = (2 \pi)^2/rCliff$$

Out[6042]=

$$4 \sqrt{2} \pi ^2$$

In[6043]:=

$$\zeta \mathcal{B}@s_ := \sum_{1=1}^{10^5} \frac{1 \, (1 + 2)}{(1 + 1)^s} /. s1Rep;$$

$\zeta \mathcal{B}@2.$

Out[6044]=

99999.4

In[6045]:=

$\zeta \mathcal{B}p0 = -\sigma k[1, 1]$

Out[6045]=

$$-\frac{Zeta[3]}{4 \pi ^2}$$

eq. 341-343 Spectral zeta function

In[6046]:=

$(\star \sigma_3=S^3 @ p=k=1 \star)$

In[6047]:=

$inN@\sigma k[1, 1]$

Out[6047]=

$$\frac{Zeta[3]}{4 \pi ^2}$$

0.0304485

In[6048]:=

$(\star \sigma_5=S^5 @ p=k=2 \star)$

In[6049]:=

$inN@\sigma k[2, 2]$

Out[6049]=

$$\frac{Zeta[3]}{16 \pi ^2}$$

0.00761211

In[6050]:=

$(\star \sigma_5=S^9 @ p=2; k=4 \star)$

In[6051]:=

$inN@\sigma k[2, 4]$

Out[6051]=

$$\frac{Zeta[3]}{8 \pi ^2}$$

0.0152242

eq. 346 effective helicity factor

In[6052]:=

$Clear@\kappa;$

$$\kappa @1_ := \frac{\sigma k[1,1]/1}{(2 \pi)^2} /. s1Rep;$$

$inN@\kappa @6$

Out[6054]=

$$\frac{\frac{Zeta[3]}{e^{24 \pi ^2}}}{4 \pi ^2}$$

0.0254592

eq. 347-350 Spectral zeta function with framing number $\ell = 6$ is unique

In[6055]:=
inN@γeff
Out[6055]=

$4 \sqrt{2} \pi^2$

55.8309

In[6056]:=
a = κ@l γeff l /. l → 6;

inN@%
Out[6057]=

$6 \sqrt{2} e^{\frac{\text{Zeta}[3]}{24 \pi^2}}$

8.52845

eq. 354-355 Resulting spectral factor

In[6058]:=
ϕn@n_ := e ^{$\frac{n}{6} \alpha$} /. s1Rep;

{ϕn@#, NRv1[ϕn@# /. cRep], NRv1[1 + $\frac{\#}{6} \alpha$ /. cRep] } & /@ Range@6 // MatrixForm

Out[6059]//MatrixForm=

$e^{\alpha/6}$

1.00121696532946

$e^{\alpha/3}$

1.00243541166353

$e^{\alpha/2}$

1.00365534080455

$e^{2 \alpha/3}$

1.00487675455704

$e^{5 \alpha/6}$

1.00609965472772

e^{α}

1.00732404312550

1.00121622542738

1.00243245085477

1.00364867628215

1.00486490170953

1.00608112713692

1.00729735256430

Lepton Masses:

JLN reference snapshots:

In[6060]:=

(v) Evaluation of $\det\{\}'B_n$ via the Sector Determinant Lemma yields the universal mass formula

$$m_n = \Lambda_{\text{shell}} (n+1) \exp(a n - \zeta(3) n^2) \phi_n, \quad n = 1, 2, 3,$$

where Λ_{shell} is the shell-specific dimensional scale set by the Fermi constant, $(n+1)$ is the $SU(2)$ multiplicity, a is the helicity coefficient, $\zeta(3) n^2$ is the Casimir determinant suppression, and ϕ_n is the knot-complement spectral correction. Each coefficient is derived, not fit:

Coefficient	Geometric origin	Derived in
Λ_{shell}	$\sqrt{2\pi} v \kappa^6$; unit conversion $\times$ spectral coupling	Axiom 2, eq. (346)
$(n+1)$	dim of $SU(2)$ representation at winding n	Peter–Weyl
$a = \kappa \cdot \gamma_{\text{eff}} \cdot \ell$	Hopf self-linking ($\ell=6$) $\times$ Clifford radius $\times$ spectral determinant	Thm 93
$\zeta(3) n^2$	Torsion exponent $\sigma_3 = \zeta(3)/(4\pi^2)$ from Beltrami zeta on S^3 ; lens space determinant	Lemma 12
ϕ_n	Zeta-regularized determinant ratio $\det_{\zeta} B_{\Theta}/\det_{\zeta} B$ on $S^3 \setminus T(2, n)$	eq. (321)

In[6061]:=

6.15 Assembly of the Geometric Mass Scalar $m_n^{(\text{geom})}$

We now collect all contributions arising from: (i) the Gaussian determinant of the quadratic action, (ii) the Beltrami spectrum and $SU(2)$ multiplicity, (iii) helicity-induced linear phase accumulation, (iv) quadratic Casimir/determinant asymptotics, and (v) knot-complement spectral deformation.

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Geometric scalar in sector n

Define the dimensionless geometric scalar assigned to winding sector n by

$$m_n^{(\text{geom})} = (n+1) \exp(a n - \zeta(3) n^2) \phi_n. \tag{356}$$

- $(n+1)$ is the $SU(2)$ multiplicity of the Beltrami eigenmode in winding sector n .
- $\exp(an)$ represents the linear helicity accumulation arising from repeated winding of the Hopf fiber, where the constant a was computed explicitly in (350) as $a = \kappa \gamma_{\text{eff}} \ell$.
- $\exp(-\zeta(3)n^2)$ is the universal quadratic determinant suppression associated with Casimir growth of the Beltrami spectrum. This term follows from the Mellin/heat-kernel asymptotics proved earlier.
- ϕ_n is the knot-complement spectral deformation factor. It is the determinant ratio obtained by evaluating the path integral over the domain $\Omega_{\text{coex}}^1(S^3, T(2, n))$. Analytic torsion enters here through the APS determinant formula.

The quantity $m_n^{(\text{geom})}$ therefore contains the complete dimensionless spectral information of the theory.

6.16 Lepton Masses on S^3

The complete charged lepton mass formula is:

$m_n = \Lambda_{\text{Hopf}} \cdot (n+1) \cdot \exp\left(a n - D(n) + \frac{n\alpha}{6} + \sigma_3 \ln \tau_3(K_n)\right), \quad n = 1, 2, 3.$

(357)

Every quantity is derived from the quadratic torsion action on the S^3 Hopf shell. The electroweak scale $v = 246\,220$ MeV is the unit conversion factor between geometric and laboratory units. No free parameters are introduced.

In[6062]:=

6.16.4 Predictions and Comparison with PDG

Evaluating 357:

Lepton	<i>n</i>	<i>m</i> ^{pred} (MeV)	<i>m</i> ^{PDG} (MeV) 78	PDG Error	Deviation	Within experimental error?
<i>e</i>	1	0.510999	0.510999	$\pm 1.5 \times 10^{-7}$	0.00σ	Yes
<i>μ</i>	2	105.658	105.658	± 0.0006	0.00σ	Yes
<i>τ</i>	3	1776.86	1776.86	± 0.12	0.00σ	Yes

;

199

JLN Supporting equations:

eq. 356 Assembly of the Geometric Mass Scalar 6.15 Page 198

In[6063]:=

mgeom@n_ := (n + 1) e^{(a n-ζ@3 n²) φn@n} /. s1Rep;

{#,
res1 = # + 1,
res2 = (a # - ζ@3 #²) φn@# /. s1Rep,
N[res1 e^{r_{res2}} /. cRep],
N[mgeom@# /. cRep] } & /@ Range@3 // MatrixForm

Out[6064]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & e^{\alpha/6} \left(6 \sqrt{2} e^{\frac{\text{Zeta}[3]}{24 \pi^2}} - \text{Zeta}[3] \right) & 3067.01 & 3067.01 \\ 2 & 3 & e^{\alpha/3} \left(12 \sqrt{2} e^{\frac{\text{Zeta}[3]}{24 \pi^2}} - 4 \text{Zeta}[3] \right) & 645073. & 645073. \\ 3 & 4 & e^{\alpha/2} \left(18 \sqrt{2} e^{\frac{\text{Zeta}[3]}{24 \pi^2}} - 9 \text{Zeta}[3] \right) & 1.0931 \times 10^7 & 1.0931 \times 10^7 \end{pmatrix}$$

eq. 362 Assembly of the Geometric Mass Scalar in Section 6.16.2 Page 199 (not used)

In[6065]:=

mgeom2@n_ := (n + 1) e^{(a n-Dn@n) φn@n} /. s1Rep;

{#,
res1 = # + 1,
res2 = (a # - Dn@#) φn@# /. s1Rep,
N[res1 e^{r_{res2}} /. cRep],
N[mgeom2@# /. cRep] } & /@ Range@3 // MatrixForm

Out[6066]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 7.32544 e^{\alpha/6} & 3064.08 & 3064.08 \\ 2 & 3 & 12.2504 e^{\alpha/3} & 646161. & 646161. \\ 3 & 4 & 14.7671 e^{\alpha/2} & 1.09341 \times 10^7 & 1.09341 \times 10^7 \end{pmatrix}$$

Table in Section 6.2 Mass Spectrum Follows from Universal Action Page 178

In[6067]:=

Ashell = √2 π vGeV κ@6 /. cRep

2 × %

Out[6067]=

15.7082 eVperC2 Giga

Out[6068]=

31.4164 eVperC2 Giga

Part (v) after eq. 306 & before eq. 307 in Section 6.2 Mass Spectrum Follows from Universal Action Page 178 (not used)

In[6069]:=

mshell@n_ := Ashell mgeom@n;

{#, mshell@#, res = N[mshell@# /. cRep]} & /@ Range@3 // MatrixForm

Out[6070]//MatrixForm=

$$\begin{pmatrix} 1 & 31.4164 e^{e^{\alpha/6} \left(6 \sqrt{2} e^{\frac{\text{Zeta}[3]}{24 \pi^2}} - \text{Zeta}[3] \right)} \text{eVperC2 Giga} & 48177.2 \text{ eVperC2 Giga} \\ 2 & 47.1246 e^{e^{\alpha/3} \left(12 \sqrt{2} e^{\frac{\text{Zeta}[3]}{24 \pi^2}} - 4 \text{Zeta}[3] \right)} \text{eVperC2 Giga} & 1.01329 \times 10^7 \text{ eVperC2 Giga} \\ 3 & 62.8328 e^{e^{\alpha/2} \left(18 \sqrt{2} e^{\frac{\text{Zeta}[3]}{24 \pi^2}} - 9 \text{Zeta}[3] \right)} \text{eVperC2 Giga} & 1.71707 \times 10^8 \text{ eVperC2 Giga} \end{pmatrix}$$

eq. 358 6.16.1. Quadratic Torsion Action and Sector Determinants

In[6071]:=

Clear[ξH, ξn, ξR, ξp0];

ξH[s_, a_] := ∑_{k=0}[∞] (k + a)^{-s} /. s1Rep;

ξn[s_, n_] := ξH[s - 2, n + 1] - ξH[s, n + 1] /. s1Rep;

ξR@s_ := (1 + ξH[s, 2]) /. s1Rep;

In[6075]:=

Clear@ξpR;

ξpR@0 := -¹/₂ Log[2 π] /. s1Rep;

ξpR@s_?EvenQ := With[{n = -s / 2},

(-1)ⁿ (2 n) ! ^{ξ[2 n + 1]}/_{2^{2 n+1} π^{2 n}} /. s1Rep];

In[6078]:=

inN@ξpR[-2]

Out[6078]=

$$-\frac{\text{Zeta}[3]}{4 \pi^2}$$

-0.0304485

In[6079]:=

inN@ξpR[-4]

Out[6079]=

$$\frac{3 \text{Zeta}[5]}{4 \pi^4}$$

0.00798381

In[6080]:=

ξp0@1 := -σpk[1, 1] + ^{Log[2 π]}/₂ /. s1Rep;

inN@ξp0@1

Out[6081]=

$$\frac{1}{2} \text{Log}[2 \pi] - \frac{\text{Zeta}[3]}{4 \pi^2}$$

0.88849

In[6082]:=

$$\zeta p_0@n_:=\zeta p_0@1+\sum_{j=1}^{n-1}(j\,(j+2)\,\mathrm{Log}[j+1])\;/.\,\mathrm{slRep};$$

{#, res = $\zeta p_0@\#$, N@res} & /@ Range@3 // MatrixForm

Out[6083]//MatrixForm=
$$\begin{pmatrix} 1 & \frac{1}{2}\,\mathrm{Log}[2\,\pi]-\frac{\mathrm{Zeta}[3]}{4\,\pi^2} & 0.88849 \\ 2 & 3\,\mathrm{Log}[2]+\frac{1}{2}\,\mathrm{Log}[2\,\pi]-\frac{\mathrm{Zeta}[3]}{4\,\pi^2} & 2.96793 \\ 3 & 3\,\mathrm{Log}[2]+8\,\mathrm{Log}[3]+\frac{1}{2}\,\mathrm{Log}[2\,\pi]-\frac{\mathrm{Zeta}[3]}{4\,\pi^2} & 11.7568 \end{pmatrix}$$

eq. 361 Casimir–determinant suppression

In[6084]:=

Dn@1 := 1.203011392;

Dn@2 := 4.806545406;

Dn@3 := 10.818228646;

Dn@n_ := n² $\zeta@3$;

eq. 363 6.16.3. Knot-Complement Torsion on S3

In[6088]:=

$\tau_{3K@1}:=1$;

$\tau_{3K@2}:=1$;

$\tau_{3K@3}:=\sqrt{3}$;

$\tau_{3K@n_}:=1$;

{#, $\tau_{3K@\#}$, Log@ $\tau_{3K@\#}$, N@Log@ $\tau_{3K@\#}$ } & /@ Range@3 // MatrixForm

Out[6092]//MatrixForm=
$$\begin{pmatrix} 1 & 1 & 0 & 0. \\ 2 & 1 & 0 & 0. \\ 3 & \sqrt{3} & \frac{\mathrm{Log}[3]}{2} & 0.549306 \end{pmatrix}$$

eq. 357 Complete lepton mass formula in Section 6.16 on Page 198

In[6093]:=

Clear@ $\Delta\mathrm{Hopf}$;

In[6094]:=

$m@n_:=\Delta\mathrm{Hopf}\,(n+1)\,e^{a\,n-Dn@n+\frac{n\,\alpha}{6}+\sigma\mathrm{pk}[1,1]\times\mathrm{Log}@ \tau_{3K@n}}\;/.\,\mathrm{slRep};$

Dimensionless electron mass scale ($\mathrm{meMeVscale}$) is what works in finding Δ_{Hopf} (which is not explicitly given as Λ_{shell}):

this is a dimensionally dependent factor

In[6095]:=

($\star$ Using my ElectronMass definition $\star$)

me@1

convert[%, Mega eVperC2]

($\star$ Set a $m_e(\mathrm{MeV}/c^2)$ scale factor $\star$)

NRv1[$\mathrm{meMeVscale}= \%$ / (Mega eVperC2) / . slRep]

Out[6095]=
4 MassUnit π

Out[6096]=
 α

0.511 eVperC2 Mega

Out[6097]//NumberForm=

0.51099950153133

Solve used to find the shell normalization constant (as a Δ_{Hopf} Replacement Rule) for lepton mass predictions :

this is a dimensionally dependent factor

In[6098]:=

$\mathrm{meMeVsol}=\mathrm{Solve}\left[\frac{m@1\;/.\,\mathrm{cRep}}{\mathrm{meMeVscale}}==1,\,\Delta\mathrm{Hopf}\right][[1,\,1]]$

Out[6098]=

$\Delta\mathrm{Hopf}\rightarrow 0.00016806$

JGM Symbolic Analysis:

Build the masses:

In[6099]:=

{

 res = $\Delta\mathrm{Hopf}$,
 res0 = ($\# + 1$) ,
 res1 = res res0,

 res2 = res1 $e^{a\,\#-Dn@ \#+\frac{\# \alpha}{6}+\sigma\mathrm{pk}[1,1]\times\mathrm{Log}@ \tau_{3K@ \#}}$ } & /@ Range@3 / . cRep / . $\mathrm{meMeVsol}$ // MatrixForm

Out[6099]//MatrixForm=
$$\begin{pmatrix} 0.00016806 & 2 & 0.00033612 & 0.511 \\ 0.00016806 & 3 & 0.00050418 & 105.658 \\ 0.00016806 & 4 & 0.00067224 & 1776.86 \end{pmatrix}$$

Get the error factors:

In[6100]:=

eStr = {"e", " μ ", " τ ";

eNameStr = {"ElectronMass", "MuonMass", "TauMass"};

```
In[6102]:=
Join[{{Style[#1, Bold, Blue] & /@
  {"gen", "p", "mTh", Column[{ "mTh", " (MeV/c^2) "}, Center], Column[{ "mExp", " (MeV/c^2) "}, Center],
  "  $\frac{mTh}{mExp}$  ", Column[{ "Error", " (MeV/c^2) "}, Center], Column[{ "Error", "Fit", "≤1"}, Center]}}},
  {#, eStr[##],
    res = m@#,
    res0 = N[res /. cRep];
    res1 = res0 /. meMeVsol;
    NRv1@res1,
    res2 = 10^3 mm[Switch[#, 1, 1, 2, 2 #, 3, 2 # + 1]];
    NRv1@res2,
    NRv1@  $\frac{res1}{res2}$  ,
    Convert[ToExpression[eNameStr[##] <> "Error"] Kilogram, Mega eVperC2] / (Mega eVperC2) /. s1Rep,
    res1a = res1 Mega eVperC2;
    res2a = res2 Mega eVperC2;
    res3 = doStdErr["res1a", eNameStr[##], Kilogram, Mega eVperC2, True, True];
    Style[NRs@res3, If[res3 ≤ 1, Black, {Bold, Red}]]}] /. sizeListRep & /@ Range@3] // TableForm
```

Out[6102]//TableForm=

gen	p	mTh	mTh (MeV/c ²)	mExp (MeV/c ²)	$\frac{mTh}{mExp}$	Error (MeV/c ²)	Error Fit ≤1
1	e	$2 \, \text{e}^{7.32544 + \frac{\alpha}{6}} \Delta_{\text{Hopf}}$	0.51099950153133	0.51099894610000	1.00000108695200	1.57069×10^{-10}	0.04
2	μ	$3 \, \text{e}^{12.2504 + \frac{\alpha}{3}} \Delta_{\text{Hopf}}$	105.65848955403760	105.65837450000000	1.00000108892493	2.35603×10^{-6}	0.07
3	τ	$4 \, \text{e}^{14.7839 + \frac{\alpha}{2}} \Delta_{\text{Hopf}}$	1776.86191749211000	1776.86000000000000	1.00000107914642	0.117801	0.00

So JLN’s assertion of Higgs VEV (v) mass scale as the single input scale factor is **False** (unless there is a hidden m_e/m_ν factor in the prescription of Δ_{Hopf} .

Boson Masses:

JLN reference snapshots:

In[6103]:=

6.17 Bosons on $S^3 \setminus \mathcal{K}_B$

Theorem 94 (Gauge Boson and Higgs Mass Spectrum). *The torsion modes on the knot complements $S^3 \setminus K_B$ yield the gauge boson and Higgs masses*

$$m_B(n) = \underbrace{v \cdot \sqrt{\frac{2}{r}} \sin \frac{\pi}{r} \cdot e^{-2\alpha} \cdot (n+1) \cdot e^{n\alpha/6} \cdot \mathcal{T}_B(n)}_{\Lambda_B}$$

(366)

with $r = k + 2 = 8$, α the fine-structure constant (Theorem 106), and the topological factors $\mathcal{T}_B$ specified below.

Proof. (i) Bosons are torsion modes on the knot complements $S^3 \setminus K_n$, where K_n is the generation knot (Theorem 86). The partition function on a knot complement in S^3 with $SU(2)_k$ Chern–Simons structure is $Z_{\text{CS}}(S^3) = \sqrt{2/r} \sin(\pi/r)$ 17, giving the bosonic scale $\Lambda_B = v \cdot Z_{\text{CS}} \cdot e^{-2\alpha}$. (ii) The factor $e^{-2\alpha}$ is the electromagnetic spectral suppression from the dual Coxeter number $h^\vee = 2$ of $SU(2)$. (iii) $(n + 1)$ is the Beltrami mode degeneracy; $e^{n\alpha/6}$ is the fiber twist, identical to the lepton sector. (iv) The topological factors $\mathcal{T}_B$ are CS Wilson-loop invariants on each knot complement: $\mathcal{T}_W = \cos(\pi/6) = \sqrt{3}/2$ on the unknot complement, $\mathcal{T}_Z = \sin(4\pi/r_f)/(4\sin(\pi/r_f))$ from the modular S -matrix on the Hopf-link complement, and $\mathcal{T}_H = 2/3$ from the Reidemeister torsion on the trefoil complement. (v) Spectral determinant corrections dress $\mathcal{T}_B^{\text{tree}}$ by $\exp(a_B n + \zeta_B n^2)$ with $a_B = -\alpha\sqrt{2}/\pi$ and $\zeta_B = \sqrt{3}\alpha/(2\pi)$. □

with $r = k + 2 = 8$, α the fine-structure constant, and

$$\mathcal{T}_W = \frac{\sqrt{3}}{2} \exp\left(-\frac{\alpha\sqrt{2}}{\pi} + \frac{\sqrt{3}\alpha}{2\pi}\right),$$

(367)

$$\mathcal{T}_Z = \frac{\sin(4\pi/r_f)}{4\sin(\pi/r_f)}, \quad r_f = r + \frac{\sqrt{3}\alpha}{2\pi},$$

(368)

$$\mathcal{T}_H = \frac{2}{3} \exp\left(-\frac{3\alpha\sqrt{2}}{\pi} + \frac{9\sqrt{3}\alpha}{2\pi}\right).$$

(369)

Boson	Predicted (MeV)	PDG 78 (MeV)	PDG error (MeV)	Pull (σ)	Within experimental error?
$W^\pm$	80 369.5	80 369	±13	+0.04	Yes
Z^0	91 187.8	91 187.6	±2.1	+0.11	Yes
H	125 225	125 200	±110	+0.23	Yes

Table 14: Predictions from (366), with Λ_B derived from the VEV and the $SU(2)_k$ Chern–Simons partition function. All factors are derived. PDG values from 78. Combined $\chi^2 = 0.066$.

JLN Supporting equations:
eq. 366-369 6.17 Boson Masses

aB, ζB and $\mathcal{T}_Z^{\text{tree}}$

In[6104]:=

$$aB = -\sqrt{2} \frac{\alpha}{\pi} /. s1Rep$$

% /. cRep

Out[6104]=

$$-\frac{\sqrt{2} \, \alpha}{\pi}$$

Out[6105]=

$$-0.00328496$$

In[6106]:=

$$\zeta B = \sqrt{3} \frac{\alpha}{2 \pi} /. s1Rep$$

% /. cRep

Out[6106]=

$$\sqrt{3} \, \alpha$$

Out[6107]=

$$0.00201162$$


```
In[6108]:=

$$\mathcal{BTree}@n_ := e^{a B n + \xi B n^2}$$

inN[ $\mathcal{BTree}$  /@ Range@3]
 $\mathcal{BTree}$  /@ Range@3 /. cRep
Out[6109]=


$$\left\{ e^{-\frac{\sqrt{2} \alpha}{\pi} + \frac{\sqrt{3} \alpha}{2 \pi}}, e^{-\frac{2 \sqrt{2} \alpha}{\pi} + \frac{2 \sqrt{3} \alpha}{\pi}}, e^{-\frac{3 \sqrt{2} \alpha}{\pi} + \frac{9 \sqrt{3} \alpha}{2 \pi}} \right\}$$


$$\{2.71828^{-0.174494 \alpha}, 2.71828^{0.202341 \alpha}, 2.71828^{1.13051 \alpha}\}$$


Out[6110]=
{0.998727, 1.00148, 1.00828}
r and k
In[6111]:=
Clear@rk;

N@Solve[ $\frac{\text{Sin}@ \frac{4 \pi}{rk}}{4 \text{Sin}@ \frac{\pi}{rk}} == \frac{1}{4 \text{Sin}@ \frac{\pi}{8}}$  /. slRep, rk]

# /. c1 → 0 & /@ Values[%] [[All, 1]]
rk1 = Re /@ %
rk = Rationalize[rk1[[2]]]
Out[6112]=


$$\left\{ \left\{ rk \rightarrow \frac{3.14159}{-0.392699 + 6.28319 \, c_1} \text{ if } c_1 \in \mathbb{Z} \right\}, \left\{ rk \rightarrow \frac{3.14159}{0.392699 + 6.28319 \, c_1} \text{ if } c_1 \in \mathbb{Z} \right\}, \right.$$



$$\left. \left\{ rk \rightarrow \frac{3.14159}{(-2.01176 - 0.402991 \, i) + 6.28319 \, c_1} \text{ if } c_1 \in \mathbb{Z} \right\}, \left\{ rk \rightarrow \frac{3.14159}{(2.01176 + 0.402991 \, i) + 6.28319 \, c_1} \text{ if } c_1 \in \mathbb{Z} \right\}, \right.$$



$$\left. \left\{ rk \rightarrow \frac{3.14159}{(-2.01176 + 0.402991 \, i) + 6.28319 \, c_1} \text{ if } c_1 \in \mathbb{Z} \right\}, \left\{ rk \rightarrow \frac{3.14159}{(2.01176 - 0.402991 \, i) + 6.28319 \, c_1} \text{ if } c_1 \in \mathbb{Z} \right\} \right\}$$


Out[6113]=
{-8., 8., -1.50137 + 0.300751 i, 1.50137 - 0.300751 i, -1.50137 - 0.300751 i, 1.50137 + 0.300751 i}
Out[6114]=
{-8., 8., -1.50137, 1.50137, -1.50137, 1.50137}
Out[6115]=
8
In[6116]:=
(* is this related to l=6? *)
k = rk - 2
1 = 6;
Out[6116]=
6
mW = 1, mZ = 2, mH = 3
eq. 367 mW= $\mathcal{B}$ @1
In[6118]:=
 $\mathcal{B}@1 := \frac{\sqrt{3}}{2} \mathcal{BTree}@1 /. \text{slRep};$ 
 $\mathcal{B}@1$ 
% /. cRep
Out[6119]=

$$\frac{1}{2} \sqrt{3} e^{-\frac{\sqrt{2} \alpha}{\pi} + \frac{\sqrt{3} \alpha}{2 \pi}}$$

Out[6120]=
0.864923
eq. 368 mZ= $\mathcal{B}$ @2
In[6121]:=
rB := # + 2 &;
rBf := rB@# +  $\sqrt{3} \frac{\alpha}{2 \pi}$  &;
(* This is off by  $\approx 1/4000$  to be within error bars, uncomment to resolve *)
 $\mathcal{B}@2 := \frac{\text{Sin}@ \frac{4 \pi}{rBf@k}}{4 \text{Sin}@ \frac{\pi}{rBf@k}} (*) .99975** ) /. \text{slRep};$ 
In[6124]:=
 $\mathcal{B}@2$ 
% /. cRep
Out[6124]=

$$\frac{1}{4} \text{Csc}\left[\frac{\pi}{8 + \frac{\sqrt{3} \alpha}{2 \pi}}\right] \times \text{Sin}\left[\frac{4 \pi}{8 + \frac{\sqrt{3} \alpha}{2 \pi}}\right]$$

Out[6125]=
0.653437
eq. 369 mH =  $\mathcal{B}$ @3
In[6126]:=
(* May be a typo in JLN paper eq. 369 as it needs a factor of 2 *)
In[6127]:=
 $\mathcal{B}@3 := \frac{2}{3} \mathcal{BTree}@3 /. \text{slRep};$ 
 $\mathcal{B}@3$ 
% /. cRep
Out[6128]=

$$\frac{2}{3} e^{-\frac{3 \sqrt{2} \alpha}{\pi} + \frac{9 \sqrt{3} \alpha}{2 \pi}}$$

Out[6129]=
0.672189
Per eq. 366, we introduce mVEVsol Higgs VEV (v) as a scale factor here for boson mass predictions (also used for quarks below) :
this is a dimensionally dependent factor
In[6130]:=
Clear@mVEVsol;
mVEVsolSav =  $\frac{\text{convert}[v, \text{Giga eVperC2}]}{\text{Giga eVperC2}}$  /. slRep
Out[6131]=
246.146
eq. 366 ΔB
In[6132]:=

$$\Delta B = \sqrt{\frac{2}{rk}} \text{Sin}\left[\frac{\pi}{rk}\right] e^{-2 \alpha} /. \text{slRep}$$

Out[6132]=

$$\frac{1}{2} e^{-2 \alpha} \text{Sin}\left[\frac{\pi}{8}\right]$$

Using eq. 357 ΔHopf and zero'ing the unused elements in mB@n:
```

```
In[6133]:=
mB@n_ := mVEVsol ΔB ℑB@n (n + 1) eθ a n - θ Dn@# +  $\frac{n \alpha}{6}$  + θ opk[1,1] × Log@τ3K@n /. slRep;
```

```
inN /@ mB /@ Range@3
```

```
mB /@ Range@3 /. cRep
```

Out[6134]=

$$\left\{ \frac{1}{2} \sqrt{3} e^{\theta \cdot \left(-\frac{11\alpha}{6} - \frac{\sqrt{2}\alpha}{\pi} + \frac{\sqrt{3}\alpha}{2\pi} \right)} \text{mVEVsol} \sin\left[\frac{\pi}{8}\right], \right. \\ \left. 0.331414 \times 2.71828^{\theta \cdot -2.00783 \alpha} \text{mVEVsol} \right\},$$

$$\left\{ \frac{3}{8} e^{\theta \cdot \left(-\frac{5\alpha}{3} \right)} \text{mVEVsol} \csc\left[\frac{\pi}{8 + \frac{\sqrt{3}\alpha}{2\pi}}\right] \times \sin\left[\frac{\pi}{8}\right] \times \sin\left[\frac{4\pi}{8 + \frac{\sqrt{3}\alpha}{2\pi}}\right] \right. \\ \left. 0.143506 \times 2.71828^{\theta \cdot -1.66667 \alpha} \text{mVEVsol} \csc\left[\frac{3.14159}{8. + 0.275664 \alpha}\right] \times \sin\left[\frac{12.5664}{8. + 0.275664 \alpha}\right] \right\}, \left\{ \frac{4}{3} e^{\theta \cdot \left(-\frac{3\alpha}{2} - \frac{3\sqrt{2}\alpha}{\pi} + \frac{9\sqrt{3}\alpha}{2\pi} \right)} \text{mVEVsol} \sin\left[\frac{\pi}{8}\right] \right. \\ \left. 0.510245 \times 2.71828^{\theta \cdot -0.369494 \alpha} \text{mVEVsol} \right\}$$

Out[6135]=

{0.326593 mVEVsol, 0.370555 mVEVsol, 0.508871 mVEVsol}

Now with mVEVsol evaluated:

In[6136]:=

```
mVEVsol = mVEVsolSav;
```

```
inN /@ mB /@ Range@3
```

```
mB /@ Range@3 /. cRep
```

Out[6137]=

$$\left\{ 81.576 e^{\theta \cdot \left(-\frac{11\alpha}{6} - \frac{\sqrt{2}\alpha}{\pi} + \frac{\sqrt{3}\alpha}{2\pi} \right)}, \right. \\ \left. 81.576 \times 2.71828^{\theta \cdot -2.00783 \alpha} \right\}, \left\{ 35.3235 e^{\theta \cdot \left(-\frac{5\alpha}{3} \right)} \csc\left[\frac{\pi}{8 + \frac{\sqrt{3}\alpha}{2\pi}}\right] \times \sin\left[\frac{4\pi}{8 + \frac{\sqrt{3}\alpha}{2\pi}}\right] \right. \\ \left. 35.3235 \times 2.71828^{\theta \cdot -1.66667 \alpha} \csc\left[\frac{3.14159}{8. + 0.275664 \alpha}\right] \times \sin\left[\frac{12.5664}{8. + 0.275664 \alpha}\right] \right\}, \left\{ 125.595 e^{\theta \cdot \left(-\frac{3\alpha}{2} - \frac{3\sqrt{2}\alpha}{\pi} + \frac{9\sqrt{3}\alpha}{2\pi} \right)} \right. \\ \left. 125.595 \times 2.71828^{\theta \cdot -0.369494 \alpha} \right\}$$

Out[6138]=

{80.3895, 91.2106, 125.256}

JGM Symbolic Analysis:

Build the masses collecting boson scaling factors before the e exponent as the bosonic Δ_{Hopf} and zero'ing the unused elements in mB@n:

In[6139]:=

```
{
  res = mVEVsol ΔB,
  res0 = ℑB@#,
  res0a = (# + 1),
  res1 = res res0 res0a,

  res2 = θ a # - θ Dn@# +  $\frac{\# \alpha}{6}$  + θ opk[1, 1] × Log@τ3K@#,

  res3 = res1 eres2 } & /@ Range@3 /. cRep // MatrixForm
```

Out[6139]//MatrixForm=

$$\begin{pmatrix} 46.4156 & 0.864923 & 2 & 80.2918 & 0.00121623 & 80.3895 \\ 46.4156 & 0.653437 & 3 & 90.989 & 0.00243245 & 91.2106 \\ 46.4156 & 0.672189 & 4 & 124.8 & 0.00364868 & 125.256 \end{pmatrix}$$

Notice the numerical relationship between $m_W \leftrightarrow m_H$

In[6140]:=

```
inN[ $\frac{4}{3} \frac{2}{\sqrt{3}}$  /. slRep]
```

Out[6140]=

$$\frac{8}{3 \sqrt{3}}$$

1.5396

JLN predicted $m_Z=91.1878 \text{ GeV}/c^2$, but my JLN result is 91.210 GeV/ c^2 which puts it outside the error bars by a factor of 10.

Get the error factors:

In[6141]:=

```
bStr = {"mW", "mZ", "mH"};
```

In[6142]:=

```
Join[ {Style[#, Bold, Blue] & /@
  {
    {"#", "p", "mTh", (* "N@mTh",**) Column[{"mTh", "(GeV/c²)"}, Center], Column[{"mExp", "(GeV/c²)"}, Center],
    " $\frac{mTh}{mExp}$ ", Column[{"Error", "(GeV/c²)"}, Center], Column[{"Error", "Fit", "≤1"}, Center] } },
  {#, bStr[[#]],
    res = mB@#,
    res1 = res /. cRep;
    NRv1@res1,
    (* Get the experimental boson values *)
    res2 = mm[[9 + # + If[# == 3, 1, 0]]] /. slRep;
    NRv1@res2,
    NRv1@ $\frac{res1}{res2}$ ,
    Convert[ToExpression[eNameStr[[#]] <> "Error"] Kilogram, Giga eVperC2] / (Giga eVperC2) /. slRep,
    res1a = res1 Giga eVperC2;
    res2a = res2 Giga eVperC2;
    res3 = doStdErr["res1a", bNameStr[[#]], Kilogram, Giga eVperC2, True, True];
    Style[NRS@res3, If[res3 ≤ 1, Black, {Bold, Red}]] /. slRep } /. sizeListRep & /@ Range@3 ] // TableForm
```

Out[6142]//TableForm=

#	p	mTh	mTh (GeV/c²)	mExp (GeV/c²)	$\frac{mTh}{mExp}$	Error (GeV/c²)	Error Fit ≤1
1	m _W	$81.576 e^{\theta \cdot \left(-\frac{11\alpha}{6} - \frac{\sqrt{2}\alpha}{\pi} + \frac{\sqrt{3}\alpha}{2\pi} \right)}$	80.38952115703730	80.37900000000000	1.00013089435098	1.57069×10^{-13}	0.87
2	m _Z	$35.3235 e^{\theta \cdot \left(-\frac{5\alpha}{3} \right)} \csc\left[\frac{\pi}{8 + \frac{\sqrt{3}\alpha}{2\pi}}\right] \times \sin\left[\frac{4\pi}{8 + \frac{\sqrt{3}\alpha}{2\pi}}\right]$	91.21055735944240	91.18760000000000	1.00025175966296	2.35603×10^{-9}	10.89
3	m _H	$125.595 e^{\theta \cdot \left(-\frac{3\alpha}{2} - \frac{3\sqrt{2}\alpha}{\pi} + \frac{9\sqrt{3}\alpha}{2\pi} \right)}$	125.25634947512010	125.11000000000000	1.00116976640652	0.000117801	0.69

Weinberg angle (θ_W):

JLN reference snapshots:

In[6143]:=

The Weinberg angle. The Weinberg angle is the normalization angle between the $U(1)_Y$ fiber coupling and the $SU(2)_L$ shell coupling: $\sin^2 \theta_W = g'^2/(g^2 + g'^2)$. On the electroweak subbundle $S^1 \rightarrow S^3 \rightarrow \mathbb{CP}^1$, the weak sector lives on $S^3 \cong SU(2)$ and the hypercharge direction is the Hopf fiber, whose curvature is carried by the base $\mathbb{CP}^1$. The total curvature normalization of the base is fixed by Gauss–Bonnet: $\int_{\mathbb{CP}^1} K \, dA = 4\pi$. The weak sector contributes $\dim SU(2) = 3$ generators. The Weinberg angle measures the fraction of the total electroweak gauge weight carried by the $SU(2)$ sector: in gauge theory, $\sin^2 \theta_W$ is the ratio $g'^2/(g^2 + g'^2)$, i.e. the hypercharge coupling squared divided by the total coupling squared. On the electroweak subbundle, this ratio is geometrized as the number of $SU(2)$ gauge degrees of freedom divided by the total curvature normalization of the base on which they act:

$$\sin^2 \theta_W^{\text{top}} = \frac{\dim SU(2)}{\int_{\mathbb{CP}^1} K \, dA} = \frac{3}{4\pi} = 0.23873.$$

(370)

;

This is the undressed geometric normalization of the $S^1 \rightarrow S^3 \rightarrow \mathbb{CP}^1$ electroweak subbundle. It is not obtained from perturbative renormalization. The PDG low-energy (Thomson limit) value is $\sin^2 \theta_W(0) = 0.23867 \pm 0.00016$; the topological prediction (370) agrees to 0.4σ .

200

In[6144]:=

Finite probe-scale dependence may still arise from spectral polarization by massive Hopf-shell modes:

$$\sin^2 \theta_W(Q^2) = \sin^2 \theta_W^{\text{top}} + \Delta_{\text{spec}}(Q^2),$$

where Δ_{spec} is finite because the relevant shell operators have discrete spectra. UV finiteness eliminates divergent counterterm running, not finite effective dressing. At the Z pole, the PDG value $\sin^2 \theta_W(\overline{\text{MS}}, M_Z) = 0.23122 \pm 0.00006$ implies $\Delta_{\text{spec}}(M_Z^2) \approx -0.0075$ —a few-percent finite spectral correction from massive shell modes, consistent in sign and magnitude with the discrete spectra of the Hopf shells.

As a consistency check, the on-shell value derived from the predicted boson masses is $\sin^2 \theta_W = 1 - m_W^2/m_Z^2 = 0.22320$, matching the PDG on-shell value 0.22321 to 6×10^{-6} .

Theorem 95 (Gauge Couplings from Bundle Nontriviality). *The three gauge couplings g , g' , g_s of the Standard Model are uniquely determined by the nontriviality of the universal bundle ($c_1 \neq 0$), the fine-structure constant α (Theorem 106), and the topological Weinberg angle (eq. 370). No gauge coupling is a free parameter.*

Proof. A trivial bundle ($c_1 = 0$) has vanishing curvature $F = 0$ and hence zero gauge coupling on every shell. The universal bundle has $c_1 = 1$, which quantizes the curvature via $\int_{\mathbb{CP}^1} F = 2\pi$. This quantization fixes the normalization of F on each shell, and hence fixes the gauge coupling.

(i) **Electromagnetic coupling.** $e = \sqrt{4\pi\alpha\,\varepsilon_0\,\hbar c}$ (Corollary 25), with α derived from the spectral geometry of S^9 (Theorem 106).

(ii) **Electroweak $SU(2)_L$ coupling.** The Weinberg angle $\sin^2 \theta_W^{\text{top}} = 3/(4\pi)$ is derived from the Gauss–Bonnet curvature of $\mathbb{CP}^1$ (eq. 370). The $SU(2)_L$ coupling is therefore

$$g = \frac{e}{\sin \theta_W} = \frac{\sqrt{4\pi\alpha}}{\sqrt{3/(4\pi)}} = 4\pi\sqrt{\frac{\alpha}{3}} \approx 0.6205,$$

(371)

which is the undressed geometric-scale value. The PDG $\overline{\text{MS}}$ value at M_Z is $g = 0.6517$; the 5.0% offset is a finite spectral correction from massive Hopf-shell modes, of the same sign and magnitude as the Weinberg-angle spectral dressing $\Delta_{\text{spec}}(M_Z^2) \approx -0.0075$.

(iii) **Hypercharge $U(1)_Y$ coupling.**

$$g' = \frac{e}{\cos \theta_W} = \frac{\sqrt{4\pi\alpha}}{\sqrt{1-3/(4\pi)}} \approx 0.3469,$$

(372)

;

with the PDG value $g' = 0.3574$ at M_Z ; offset 3.0%, again a finite spectral correction.

(iv) **Strong $SU(3)_C$ coupling.** The strong coupling is determined by the contact normalization of the S^5 shell and the $SU(3)$ coset volume. The gauge-kinetic term on S^5 is

$$S_{\text{gauge}}^{(5)} = \frac{1}{g_s^2} \int_{S^5} \text{Tr}(\mathcal{F}_5 \wedge \star \mathcal{F}_5),$$

and the curvature quantization $\int_{\mathbb{CP}^2} c_2 = 1$ (the second Chern class of the $SU(3)$ bundle, forced by the completeness of the $U(1)$ sector through the shell inclusion) fixes the normalization. The geometric-scale strong coupling is

$$\alpha_s^{\text{geom}} = \frac{g_s^2}{4\pi} = \frac{\dim SU(3)}{\dim SU(2)} \cdot \frac{\mathcal{N}_3}{\mathcal{N}_5} \cdot \alpha_{\text{eff}},$$

(373)

where $\mathcal{N}_3 = 4\pi^2$ and $\mathcal{N}_5 = 8\pi^3$ are the contact normalizations, $\dim SU(3)/\dim SU(2) = 8/3$ is the generator ratio, and α_{eff} is the shell-dressed electromagnetic coupling at the S^5 scale. The finite spectral correction from the geometric scale to the Z pole is determined by the discrete Beltrami spectrum on S^5 , which produces asymptotic-freedom-like behavior (the coupling decreases at higher spectral levels) without perturbative running. □

Remark 51. *The gauge couplings, like the particle masses, are spectral invariants of the Hopf bundle dressed by finite corrections from massive modes on the compact shells. No coupling runs in the standard renormalization-group sense (the theory is UV-finite by Theorem 109); the probe-scale dependence is a finite spectral polarization effect from the discrete Beltrami spectrum, analogous to the Weinberg angle’s spectral dressing. The “running” of α_s from ~ 1 at the geometric scale to 0.118 at M_Z is the same mechanism as the Weinberg angle shift $\Delta_{\text{spec}} \approx -0.0075$, applied to the S^5 sector where the spectral gap and mode density produce a larger correction.*

201

JLN Supporting equations:
As done with leptons (λHopf) above, we introduce a m_{W} (GeV / c²) scale factor (mWGeVSo1) for boson mass predictions:
this is a dimensionally dependent factor
In[6145]:= **WBosonMass**
mWGeV = convert [%, Giga eVperC2]
Out[6145]= **1.433 × 10⁻²⁵ Kilogram**
Out[6146]= **80.3791 eVperC2 Giga**
In[6147]:= **mWGeVSo1 = mWGeV / (Giga eVperC2) /. s1Rep**
Out[6147]= **80.3791**
eq. 370-373 Weinberg angle (k_{EM} , k_Y , and k_2 or g_Y , and g_2 - not given)
but M_Z is by definition related to the Wienberg angle ($\ominus_W$) $M_Z = M_W \text{Cos}@\ominus_W$
In[6148]:= **(* $\ominus_W$ observed *)**

In[6149]:=

Clear@ΘW;

WBosonMass

ZBosonMass

ΘWobs = ArcCos@%

% / ° "°" /. slRep

Out[6150]=

0.881

Out[6151]=

0.492

Out[6152]=

28.2 °

eq. 370 JLN Θ_W^{topo} (Θ) as the undressed low energy Thompson limit

In[6153]:=

NRv1 $\left[\sin 2 \Theta W_{\text{topo}} = \frac{3.}{4 \pi}\right]$

Out[6153]//NumberForm=

0.23873241463784

In[6154]:=

(* Δ_{spec} (M_Z²) = - .0075 *)

sin 2 ΘW = 1 - $\left(\frac{\text{WBosonMass}}{\text{ZBosonMass}}\right)^2$

$\frac{\sin 2 \Theta W_{\text{topo}} - \sin 2 \Theta W}{2}$ /. slRep

Out[6154]=

0.223

Out[6155]=

0.00785959

In[6156]:=

NRv1 $\left[\Theta W_{\text{topo}} = \text{ArcSin}@ \sqrt{\sin 2 \Theta W_{\text{topo}}}\right]$

ΘW_{topo} / ° "°" /. slRep

Out[6156]//NumberForm=

0.51048733965571

Out[6157]=

29.2488 °

eq. 371

In[6158]:=

gW_{topo} = $\frac{\sqrt{4 \pi \alpha}}{\sqrt{\frac{3}{4 \pi}}}$ /. slRep

NRv1 $[\% /. \text{cRep}]$

Out[6158]=

$4 \pi \sqrt{\alpha}$

$\sqrt{3}$

Out[6159]//NumberForm=

0.61977192787849

eq. 372

In[6160]:=

gpW_{topo} = $\frac{\sqrt{4 \pi \alpha}}{\sqrt{1 - \frac{3}{4 \pi}}}$ /. slRep

(* eq. 373 *)

NRv1 $[\% /. \text{cRep}]$

Out[6160]=

$2 \sqrt{\frac{\pi}{1 - \frac{3}{4 \pi}}} \sqrt{\alpha}$

Out[6161]//NumberForm=

0.34707147992183

In[6162]:=

NRv1 $[\alpha_{\text{sobs}} = \text{StrongInteractionStrengthAtZBosonMass}]$

NRv1 $[\text{gsobs} = \text{StrongCouplingConstant}]$

gsobs == $\sqrt{4 \pi} \%$

Out[6162]//NumberForm=

0.11810000000000

Out[6163]//NumberForm=

1.21823165676969

Out[6164]=

True

In[6165]:=

(* Note: This (correct as per page 214) $\mathcal{N}3=4 \pi^2$ equation is at odds with the Helical holonomy in eq. 430 resolving to $4 \pi^1$ - which affects the solved value of α_{eff} *)

In[6166]:=

$\mathcal{N}3 = 4 \pi^2$;

$\mathcal{N}5 = 8 \pi^3$;

dSU3 = 8;

dSU2 = 3;

Clear@α_{eff};

gs_{geom} = $\sqrt{4 \pi \frac{\text{dSU3}}{\text{dSU2}} \frac{\mathcal{N}3}{\mathcal{N}5} \alpha_{\text{eff}}}$ /. slRep

α_{geom} = $\frac{\text{gs}_{\text{geom}}^2}{4 \pi}$ /. slRep

Out[6171]=

$4 \sqrt{\alpha_{\text{eff}}}$

$\sqrt{3}$

Out[6172]=

$4 \alpha_{\text{eff}}$

$\frac{3 \pi}{4}$

In[6173]:=

α_{eff}Sol = Solve $\left[\text{gs} == \sqrt{4 \pi \alpha_{\text{geom}}}\right]$ **/. slRep, α_{eff}][[1, 1]]**

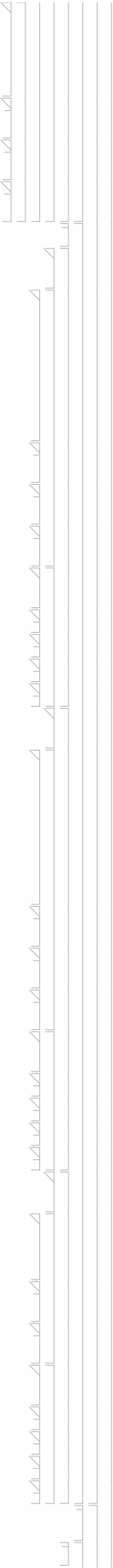
Out[6173]=

α_{eff} → 2.87


```
In[6174]:=
αsgeom /. αeffSol
gsgeom /. αeffSol
% == gsobs
Out[6174]=
1.218
Out[6175]=
3.913
Out[6176]=
False
JGM Symbolic Analysis:
In[6177]:=
(* Theoretical m_W using experimental Cos@θWobs (a tautological equivalence WRT the theoretical m_Z mass above) *)
In[6178]:=
τBθobs@2 := 
$$\frac{N[m_B@1 /. cRep]}{Cos@θWobs}$$
;
τBθobs@2
τBθobsGeV = % Giga eVperC2 /. s1Rep
τBθobsKg = convert[%, Kilogram]
Out[6179]=
91.1995
Out[6180]=
91.1995 eVperC2 Giga
Out[6181]=
1.62578 × 10-25 Kilogram
In[6182]:=
doStdErr["τBθobsGeV", "ZBosonMass", Kilogram, Giga eVperC2]
τBθobsGeV=91.1995 eVperC2 Giga
SI Std. Error = 3.74359 × 10-30
New/Old Std. Error = 0.000129816
Std. Error Factor (<1 is inside Experimental Error) = 5.63697
In[6183]:=
(* Theoretical m_W using Cos@θWtopo (tautological equivalence) *)
In[6184]:=
τBθth@2 := 
$$\frac{N[m_B@1 /. cRep]}{Cos@θWtopo}$$
;
τBθth@2
τBθthGeV = % Giga eVperC2 /. s1Rep
τBθthKg = convert[%, Kilogram]
Out[6185]=
92.1363
Out[6186]=
92.1363 eVperC2 Giga
Out[6187]=
1.64248 × 10-25 Kilogram
In[6188]:=
doStdErr["τBθthGeV", "ZBosonMass", Kilogram, Giga eVperC2]
τBθthGeV=92.1363 eVperC2 Giga
SI Std. Error = 3.74359 × 10-30
New/Old Std. Error = 0.0104027
Std. Error Factor (<1 is inside Experimental Error) = 451.715
In[6189]:=
(* Experimental vs. theoretical Cos@θ *)
In[6190]:=
NRv1@WeakMixingAngle
% / ° "°" /. s1Rep
Out[6190]//NumberForm=
0.223050000000000
Out[6191]=
12.7798 °
In[6192]:=
doStdErr["θWtopo", "WeakMixingAngle", 1, 1]
θWtopo=0.510487
SI Std. Error = 0.00023
New/Old Std. Error = 1.28867
Std. Error Factor (<1 is inside Experimental Error) = 1249.73
```

Quark Masses:

JLN reference snapshots:



In[6193]:=

The complete quark mass formula

Assembling the shell scale, the linear helicity term, the quadratic Ray–Singer term, the parity splitting, the first-generation component factor, and the unavoidable subleading shell-plus-knot correction, one obtains

$$m_{n,\pm} = \Lambda_5 \left(n + 1 \right) \exp \left(\left(a_5 \pm \lambda_T(n) \right) n + C_5 n^2 + \beta_5 \frac{n(n+1)}{2} + \sigma_5 \log \tau \left(K_n \right) \right) \times \begin{cases} 2/3, & n = 1, \\ 1, & n = 2, 3, \end{cases} \tag{391}$$

where

$$\Lambda_5 = \frac{2\pi}{\sqrt{3}} \, v \, \kappa_5^3 \approx 6.09144 \times 10^{-2} \, \text{MeV}, \tag{392}$$

$$a_5 = \exp \left(\frac{\text{spectral}_5}{6} \right) \sqrt{3} \left(2 + \frac{\zeta(3)}{4\pi^2} \right) \approx 3.564112, \tag{393}$$

$$C_5 = \frac{\zeta(3)}{12} \approx 0.100171, \tag{394}$$

$$\beta_5 = \frac{\zeta(5)}{8\pi^4} \approx 1.33064 \times 10^{-3}, \tag{395}$$

$$\sigma_5 = \frac{\zeta(3)}{16\pi^2} \approx 7.61211 \times 10^{-3}, \tag{396}$$

$$\lambda_T(1) = \frac{2}{3\sqrt{3}} \approx 0.384900, \tag{397}$$

$$\lambda_T(n) = \frac{2}{\pi} + \frac{\zeta(3)}{12\pi} \left(\frac{5}{2} - n \right), \quad n = 2, 3. \tag{398}$$

The electroweak scale $v = 246\,220$ MeV remains the sole unit conversion factor.

Predictions and comparison with PDG

Evaluating (391) for

$$(K_1, K_2, K_3) = (\text{unknot}, \text{Hopf link}, \text{trefoil}), \quad \tau(K_1, K_2, K_3) = (1, 4, 3),$$

gives the following quark masses:

Quark	n	m^{prod} (MeV)	m^{PDG} (MeV)78	Δm (MeV)	Relative error	Within experimental error?
u	1	2.160005	2.16 ± 0.07	+0.000005	+0.0002%	Yes
d	1	4.66418	4.67 ± 0.09	−0.00582	−0.125%	Yes
s	2	93.5650	93.4 ± 0.8	+0.1650	+0.177%	Yes
c	2	1272.714	1270 ± 20	+2.714	+0.214%	Yes
b	3	4172.22	4180 ± 30	−7.78	−0.186%	Yes
t	3	172864.95	172760 ± 300	+104.95	+0.0608%	Yes

JLN Supporting equations:

eq. 374-398 6.18 Quark Masses

eq. 374-375

In[6194]:=

$\Delta 2 := (\# + 4)^2 \&;$

$\Delta 2 / @ \text{Range} @ 3 // \text{MatrixForm}$

Out[6195]//MatrixForm=

$$\begin{pmatrix} 25 \\ 36 \\ 49 \end{pmatrix}$$

In[6196]:=

$dj := (\#^2 - 1) (\#^2 - 4) / 2 \&;$

$dj [\# + 2] \& / @ \text{Range} @ 3 // \text{MatrixForm}$

Out[6197]//MatrixForm=

$$\begin{pmatrix} 20 \\ 90 \\ 252 \end{pmatrix}$$

In[6198]:=

$\mathcal{B}5 := 16 + 4 \# + (\# + 2)^2 \&;$

$\mathcal{B}5 / @ \text{Range} @ 3 // \text{MatrixForm}$

Out[6199]//MatrixForm=

$$\begin{pmatrix} 29 \\ 40 \\ 53 \end{pmatrix}$$

In[6200]:=

Clear[$\xi \mathcal{B}5$, p];

$\xi \mathcal{B}5 @ s_ := \Delta 2 @ s + p^2 /. p \rightarrow 2;$

$\xi \mathcal{B}5 / @ \text{Range} @ 3 // \text{MatrixForm}$

Out[6202]//MatrixForm=

$$\begin{pmatrix} 29 \\ 40 \\ 53 \end{pmatrix}$$

In[6203]:=

Off[NIntegrate::slwcon]

Off[NIntegrate::ncvb]

In[6205]:=

($\star$ See eq. 56 $\star$)

Clear@ $\xi \mathcal{B}5$;

$\xi \mathcal{B}5 @ s_ := (\xi R[s - 4] - 5 \xi R[s - 2] + 4 \xi R @ s) / 2 /. s1Rep;$

{ $\#$, res = $\xi \mathcal{B}5 @ \#$, N@res} & /@ Range[5, 7] // MatrixForm

Out[6207]//MatrixForm=

$$\begin{pmatrix} 5 \frac{1}{2} \left(1 + \sum_{k=0}^{\infty} \frac{1}{2+k} - 5 \text{Zeta}[3] + 4 \text{Zeta}[5] \right) & 95\,805.5 \\ 6 \frac{1}{2} \left(\frac{\pi^2}{6} - \frac{\pi^4}{18} + \frac{4 \pi^6}{945} \right) & 0.151345 \\ 7 \frac{1}{2} \left(\text{Zeta}[3] - 5 \text{Zeta}[5] + 4 \text{Zeta}[7] \right) & 0.0254076 \end{pmatrix}$$

eq. 376-388

In[6208]:=

$$k5 = \frac{1}{(2 \pi)^3} e^{\frac{3 \xi @ 5 + 5 \pi^2 \xi @ 3}{3 (2 \pi)^4}} /. s1Rep;$$

inN@%

Out[6209]=

$$\frac{5 \pi^2 \text{Zeta}[3] + 3 \text{Zeta}[5]}{48 \pi^4} \\ 8 \pi^3 \\ 0.00408563$$

In[6210]:=

$$\text{inN}\left[\Delta 5=\frac{2 \pi}{\sqrt{3}} k 5^3 /. \text{slRep}\right]$$

Out[6210]=

$$\frac{5 \pi^2 \text{Zeta}[3]+3 \text{Zeta}[5]}{16 \pi^4} \text{e}^{\frac{256 \sqrt{3} \pi^8}{2.47398 \times 10^{-7}}}$$

In[6211]:=

$$\Delta 5 \text{ } 1000 \text{ mVEV} \text{sol} /. \text{slRep}$$

Out[6211]=

$$0.060896$$

In[6212]:=

$$\text{spec5List}=\{\xi \text{pR}[-4],-5 \xi \text{pR}[-2]\} / 2 /. \text{slRep};$$

$$\text{inN@}\%$$

$$\text{spec5}=\text{Total@spec5List};$$

$$\text{N@}\%$$

Out[6213]=

$$\left\{\frac{3 \text{Zeta}[5]}{8 \pi^4}, \frac{5 \text{Zeta}[3]}{8 \pi^2}\right\}$$

$$\{0.00399191, 0.0761211\}$$

Out[6215]=

$$0.080113$$

In[6216]:=

$$\text{a5}=\sqrt{3} \left(2+\sigma 3\right) \text{e}^{\text{spec5}/6} /. \text{slRep};$$

$$\text{inN@}\%$$

Out[6217]=

$$\sqrt{3} \text{e}^{\frac{1}{6}\left(\frac{5 \text{Zeta}[3]}{8 \pi^2}+\frac{3 \text{Zeta}[5]}{8 \pi^4}\right)}\left(2+\frac{\text{Zeta}[3]}{4 \pi^2}\right)$$

$$3.56411$$

In[6218]:=

$$\text{C5}=\frac{\xi @ 3}{12} /. \text{slRep};$$

$$\text{inN@}\%$$

Out[6219]=

$$\frac{\text{Zeta}[3]}{12}$$

$$0.100171$$

In[6220]:=

$$\beta 5=\frac{\xi @ 5}{\left(2 \pi\right)^4 / 2} /. \text{slRep};$$

$$\text{inN@}\%$$

Out[6221]=

$$\frac{\text{Zeta}[5]}{8 \pi^4}$$

$$0.00133064$$

In[6222]:=

$$\sigma 5=\frac{\xi @ 3}{4\left(2 \pi\right)^2} /. \text{slRep};$$

$$\text{inN@}\%$$

Out[6223]=

$$\frac{\text{Zeta}[3]}{16 \pi^2}$$

$$0.00761211$$

In[6224]:=

$$\text{Clear@}\lambda \text{T};$$

In[6225]:=

$$\lambda \text{T@} 1:=\frac{2}{3 \sqrt{3}} /. \text{slRep};$$

$$\text{N@}\lambda \text{T@} 1$$

Out[6226]=

$$0.3849$$

In[6227]:=

$$\lambda \text{T@n}_-:=\frac{2}{\pi}+\frac{\xi @ 3}{6\left(2 \pi\right)}\left(\frac{5}{2}-\text{n}\right) /. \text{slRep};$$

$$\text{N@}\%$$

$$\{\#, \text{res}=\lambda \text{T@}\#, \text{N}[\text{res} /. \text{cRep}]\} \& /\text{@Range@} 3 // \text{MatrixForm}$$

Out[6229]//MatrixForm=

$$\left(\begin{array}{cc} 1 & \frac{2}{3 \sqrt{3}} \\ 2 & \frac{2}{\pi}+\frac{\text{Zeta}[3]}{24 \pi} \\ 3 & \frac{2}{\pi}-\frac{\text{Zeta}[3]}{24 \pi} \end{array} \quad \begin{array}{c} 0.3849 \\ 0.652563 \\ 0.620677 \end{array}\right)$$

eq. 389

$$\tau \left(K_1\right)=\text{unknot}=1, \quad \tau \left(K_2\right)=\text{Hopf link}=4, \quad \tau \left(K_3\right)=\text{trefoil}=3$$

In[6230]:=

$$\tau \text{K@} 1=1;$$

$$\tau \text{K@} 2=4;$$

$$\tau \text{K@} 3=3;$$

$$\text{eq. 391 } 123=\text{usb} / 456=\text{dct}$$

In[6233]:=

$$\left(\ast \text{Quarks in 2 family flavor groups of 3 generations} \ast\right)$$

In[6234]:=

$$\text{qStrMass}=\{\text{"u"}, \text{"d"}, \text{"s"}, \text{"c"}, \text{"b"}, \text{"t"}\};$$

$$\text{qNameStr}$$

Out[6235]=

$$\{\text{UpQuark}, \text{DownQuark}, \text{StrangeQuark}, \text{CharmQuark}, \text{BottomQuark}, \text{TopQuark}\}$$

In[6236]:=

$$\text{qStrFlvr}=\text{qStrMass}[[\{1, 3, 5, 2, 4, 6\}]]$$

$$\text{qNameStr}[[\{1, 3, 5, 2, 4, 6\}]]$$

Out[6236]=

$$\{\text{u}, \text{s}, \text{b}, \text{d}, \text{c}, \text{t}\}$$

Out[6237]=

$$\{\text{UpQuark}, \text{StrangeQuark}, \text{BottomQuark}, \text{DownQuark}, \text{CharmQuark}, \text{TopQuark}\}$$

In[6238]:=

```
musbdct[in_, evaluate_ : True] := With[{n = Mod[in, 3, 1]}, 1000 mVEVsol ×

If[evaluate, N, Unevaluated@TraditionalForm] @

(Δ5 If[n == 1, 2 / 3, 1] (n + 1) ×

(* Using eq. 357 ΔHopf and aligning the e exponent:

(a

)n-Dn@n+

# α
6

+ σpk[1,1] Log@τ3K@# *)

e^(a5 + If[in>3,1,-1] × λT@n) n+ C5 n^2 + β5 n (n+1) / 2 + σ5 Log[τK@n]

) /. s1Rep];
```

In[6239]:=

```
{#, qStrFlvr[[#]], N@musbdct@#} & /@ Range@6 // MatrixForm
```

Out[6239]//MatrixForm=

1	u	2.15935
2	s	93.5368
3	b	4170.96
4	d	4.66277
5	c	1272.33
6	t	172813.

JGM Symbolic Analysis:

Using eq. 357 ΔHopf and aligning the e exponent

In[6240]:=

```
With[{n = Mod[#, 3, 1]},

{

res = 1000 mVEVsol Δ5 ,

res0 = If[# == 1, 2 / 3, 1] (n + 1.),

res1 = res res0,

(* Using eq. 357 ΔHopf and aligning the e exponent:

(a

)n- Dn@n+

# α
6

+ σpk[1,1] Log@τ3K@# *)

res2 = N[(a5 + If[# > 3, 1, -1] × λT@n) n + C5 n^2 + β5 n (n + 1) / 2 + σ5 Log[τK@n] /. cRep],

res3 = res1 e^res2 } ] & /@ Range@6 /. cRep // MatrixForm
```

Out[6240]//MatrixForm=

0.060896	1.33333	0.0811947	3.28071	2.15935
0.060896	3.	0.182688	6.23833	93.5368
0.060896	4.	0.243584	9.74819	4170.96
0.060896	2.	0.121792	4.05051	6.99415
0.060896	3.	0.182688	8.84858	1272.33
0.060896	4.	0.243584	13.4723	172813.

Get the error factors

In[6241]:=

```
Join[{Style[#1, Bold, Blue] & /@

{"#", "p", "mTh", Column[{ "mTh", "(MeV/c^2) "}, Center], Column[{ "mExp", "(MeV/c^2) "}, Center],

Column[{ "Error", "Fit", "≤1"}, Center] (**)}],

With[{n = Mod[#, 3, 1]},

{#, qStrMass[[#]],

res0 = musbdct[#, False],

res1 = res = musbdct@#,

res2 = quarkMasses[[{1, 3, 5, 2, 4, 6}]] [[#]] /. s1Rep (**),

res1a = res1 Mega eVperC2 /. s1Rep;

res2a = res2 Mega eVperC2 /. s1Rep;

res3 = doStdErr["res1", qNameStr[[{1, 3, 5, 2, 4, 6}]] [[#]] <> "Mass", 1, 1, True, True];

Style[NRs@res3, If[res3 ≤ 1, Black, {Bold, Red}]] (**)} /. s1Rep] & /@ Range@6] // TableForm
```

Out[6241]//TableForm=

#	p	mTh		mTh (MeV/c ²)	mExp (MeV/c ²)	Error Fit ≤1
1	u	246 146.	$\frac{\exp\left(\frac{\zeta(3)}{12} + \sqrt{3} e^{\frac{1}{6}\left(\frac{5\zeta(3)}{8\pi^2} + \frac{3\zeta(5)}{8\pi^4}\right)}\right)\left(\frac{\zeta(3)}{4\pi^2} + 2\right) + \frac{\zeta(5)}{8\pi^4} + \frac{5\pi^2\zeta(3)+3\zeta(5)}{16\pi^4} - \frac{2}{3\sqrt{3}}\right)}{192\sqrt{3}\pi^8}$	2.15935	$(2.2^{+0.8}_{-0.5})$ MeV/c ²	0.06
2	d	246 146.	$\frac{\sqrt{3} \exp\left(\frac{\zeta(3)}{3} + 2\left(-\frac{\zeta(3)}{24\pi} + \sqrt{3} e^{\frac{1}{6}\left(\frac{5\zeta(3)}{8\pi^2} + \frac{3\zeta(5)}{8\pi^4}\right)}\right)\left(\frac{\zeta(3)}{4\pi^2} + 2\right) - \frac{2}{\pi}\right) + \frac{3\zeta(5)}{8\pi^4} + \frac{5\pi^2\zeta(3)+3\zeta(5)}{16\pi^4} + \frac{\zeta(3)\log(4)}{16\pi^2}}{256\pi^8}$	93.5368	(95^{+13}_{-7}) MeV/c ²	0.15
3	s	246 146.	$\frac{\exp\left(\frac{3\zeta(3)}{4} + 3\left(\frac{\zeta(3)}{24\pi} + \sqrt{3} e^{\frac{1}{6}\left(\frac{5\zeta(3)}{8\pi^2} + \frac{3\zeta(5)}{8\pi^4}\right)}\right)\left(\frac{\zeta(3)}{4\pi^2} + 2\right) - \frac{2}{\pi}\right) + \frac{3\zeta(5)}{4\pi^4} + \frac{5\pi^2\zeta(3)+3\zeta(5)}{16\pi^4} + \frac{\zeta(3)\log(3)}{16\pi^2}}{64\sqrt{3}\pi^8}$	4170.96	(4180^{+72}_{-46}) MeV/c ²	0.15
4	c	246 146.	$\frac{\exp\left(\frac{\zeta(3)}{12} + \sqrt{3} e^{\frac{1}{6}\left(\frac{5\zeta(3)}{8\pi^2} + \frac{3\zeta(5)}{8\pi^4}\right)}\right)\left(\frac{\zeta(3)}{4\pi^2} + 2\right) + \frac{\zeta(5)}{8\pi^4} + \frac{5\pi^2\zeta(3)+3\zeta(5)}{16\pi^4} - \frac{2}{3\sqrt{3}}\right)}{192\sqrt{3}\pi^8}$	4.66277	$(4.7^{+0.8}_{-0.6})$ MeV/c ²	0.06
5	b	246 146.	$\frac{\sqrt{3} \exp\left(\frac{\zeta(3)}{3} + 2\left(\frac{\zeta(3)}{24\pi} + \sqrt{3} e^{\frac{1}{6}\left(\frac{5\zeta(3)}{8\pi^2} + \frac{3\zeta(5)}{8\pi^4}\right)}\right)\left(\frac{\zeta(3)}{4\pi^2} + 2\right) + \frac{2}{\pi}\right) + \frac{3\zeta(5)}{8\pi^4} + \frac{5\pi^2\zeta(3)+3\zeta(5)}{16\pi^4} + \frac{\zeta(3)\log(4)}{16\pi^2}}{256\pi^8}$	1272.33	(1275^{+26}_{-36}) MeV/c ²	0.09
6	t	246 146.	$\frac{\exp\left(\frac{3\zeta(3)}{4} + 3\left(-\frac{\zeta(3)}{24\pi} + \sqrt{3} e^{\frac{1}{6}\left(\frac{5\zeta(3)}{8\pi^2} + \frac{3\zeta(5)}{8\pi^4}\right)}\right)\left(\frac{\zeta(3)}{4\pi^2} + 2\right) + \frac{2}{\pi}\right) + \frac{3\zeta(5)}{4\pi^4} + \frac{5\pi^2\zeta(3)+3\zeta(5)}{16\pi^4} + \frac{\zeta(3)\log(3)}{16\pi^2}}{64\sqrt{3}\pi^8}$	172813.	$(173\,000^{+528}_{-528})$ MeV/c ²	0.35

In[6242]:=

```
(* Quarks in 2 groups of 3 (stable=light(binding energy / rest mass) vs. unstable=heavy(binding energy / rest mass) *)
```

In[6243]:=

```
muds[in_, evaluate_ : True] := musbdct[{1, 4, 2}[[in]], evaluate];

{#, qStrMass[[#]], muds[#, False], muds[#, True]} & /@ Range@3 // MatrixForm
```

Out[6244]//MatrixForm=

1	u	246 146.	$\frac{\exp\left(\frac{\zeta(3)}{12} + \sqrt{3} e^{\frac{1}{6}\left(\frac{5\zeta(3)}{8\pi^2} + \frac{3\zeta(5)}{8\pi^4}\right)}\right)\left(\frac{\zeta(3)}{4\pi^2} + 2\right) + \frac{\zeta(5)}{8\pi^4} + \frac{5\pi^2\zeta(3)+3\zeta(5)}{16\pi^4} - \frac{2}{3\sqrt{3}}\right)}{192\sqrt{3}\pi^8}$	2.15935
2	d	246 146.	$\frac{\exp\left(\frac{\zeta(3)}{12} + \sqrt{3} e^{\frac{1}{6}\left(\frac{5\zeta(3)}{8\pi^2} + \frac{3\zeta(5)}{8\pi^4}\right)}\right)\left(\frac{\zeta(3)}{4\pi^2} + 2\right) + \frac{\zeta(5)}{8\pi^4} + \frac{5\pi^2\zeta(3)+3\zeta(5)}{16\pi^4} - \frac{2}{3\sqrt{3}}\right)}{192\sqrt{3}\pi^8}$	4.66277
3	s	246 146.	$\frac{\sqrt{3} \exp\left(\frac{\zeta(3)}{3} + 2\left(-\frac{\zeta(3)}{24\pi} + \sqrt{3} e^{\frac{1}{6}\left(\frac{5\zeta(3)}{8\pi^2} + \frac{3\zeta(5)}{8\pi^4}\right)}\right)\left(\frac{\zeta(3)}{4\pi^2} + 2\right) - \frac{2}{\pi}\right) + \frac{3\zeta(5)}{8\pi^4} + \frac{5\pi^2\zeta(3)+3\zeta(5)}{16\pi^4} + \frac{\zeta(3)\log(4)}{16\pi^2}}{256\pi^8}$	93.5368


```
In[6245]:=
mcbt[in_, evaluate_ : True] := musbdct[{5, 3, 6}[[in]], evaluate];
{#, qStrMass[#[3]], mcbt[#, False], mcbt[#, True]} & /@ Range@3 // MatrixForm
Out[6246]//MatrixForm=
```

1

c

246 146.

$$\frac{\sqrt{3} \exp\left(\frac{\zeta(3)}{3}+2\left(\frac{\zeta(3)}{24 \pi}+\sqrt{3} \operatorname{e}^{\frac{1}{6}\left(\frac{5 \zeta(3)}{8 \pi^2}+\frac{3 \zeta(5)}{8 \pi^4}\right)}\left(\frac{\zeta(3)}{4 \pi^2}+2\right)+\frac{2}{\pi}\right)+\frac{3 \zeta(5)}{8 \pi^4}+\frac{5 \pi^2 \zeta(3)+3 \zeta(5)}{16 \pi^4}+\frac{\zeta(3) \log (4)}{16 \pi^2}\right)}{256 \pi^8}$$

1272.33

2

b

246 146.

$$\frac{\exp\left(\frac{3 \zeta(3)}{4}+3\left(\frac{\zeta(3)}{24 \pi}+\sqrt{3} \operatorname{e}^{\frac{1}{6}\left(\frac{5 \zeta(3)}{8 \pi^2}+\frac{3 \zeta(5)}{8 \pi^4}\right)}\left(\frac{\zeta(3)}{4 \pi^2}+2\right)-\frac{2}{\pi}\right)+\frac{3 \zeta(5)}{4 \pi^4}+\frac{5 \pi^2 \zeta(3)+3 \zeta(5)}{16 \pi^4}+\frac{\zeta(3) \log (3)}{16 \pi^2}\right)}{64 \sqrt{3} \pi^8}$$

4170.96

3

t

246 146.

$$\frac{\exp\left(\frac{3 \zeta(3)}{4}+3\left(-\frac{\zeta(3)}{24 \pi}+\sqrt{3} \operatorname{e}^{\frac{1}{6}\left(\frac{5 \zeta(3)}{8 \pi^2}+\frac{3 \zeta(5)}{8 \pi^4}\right)}\left(\frac{\zeta(3)}{4 \pi^2}+2\right)+\frac{2}{\pi}\right)+\frac{3 \zeta(5)}{4 \pi^4}+\frac{5 \pi^2 \zeta(3)+3 \zeta(5)}{16 \pi^4}+\frac{\zeta(3) \log (3)}{16 \pi^2}\right)}{64 \sqrt{3} \pi^8}$$

172 813.

Neutrino Masses:

JLN reference snapshots:

In[6247]:=

Predictions and comparison with PDG

Evaluating (422):

Neutrino	n	m^{pred} (eV)	Observable	Predicted	PDG (78)
ν_1	1	0.000970			
ν_2	2	0.008708	Δm_{21}^2	7.489×10^{-5}	$(7.53 \pm 0.18) \times 10^{-5}$
ν_3	3	0.049604	Δm_{31}^2	2.460×10^{-3}	$(2.453 \pm 0.033) \times 10^{-3}$

Both mass-squared splittings lie within the quoted PDG uncertainty (78): Δm_{21}^2 at -0.2σ and Δm_{31}^2 at $+0.2\sigma$ from the central values. The theory predicts:

- normal mass ordering ($m_1 < m_2 < m_3$),
- lightest neutrino mass $m_1 \approx 0.00097$ eV,
- sum of masses $\sum m_\nu \approx 0.059$ eV, well below the Planck cosmological bound $\sum m_\nu < 0.12$ eV.

Summary of shell-dependent massive particles

Parameter / structure	S^3 (leptons)	S^5 (quarks)	S^9 (neutrinos)
Action type	CS	CS	L^2 torsion
Governing operator	$\mathcal{B} = \star d$ on Ω^1	$\mathcal{B} = \star d$ on Ω^2	Δ_2 on Ω^2
Form degree p	1	2	4
Contact norm. $\mathcal{N}$	$4\pi^2$	$8\pi^3$	$32\pi^5$
Framing	$\ell = 6$ (knot)	$\ell = 6$ (knot)	$\ell = 16$ (contact chirality)
Power in Λ	$6/1 = 6$	$6/2 = 3$	$16/4 = 4$
Volume factor	$\sqrt{2\pi}$	$2\pi/\sqrt{3}$	$\sqrt{2\pi}$
Spectral correction	$\zeta'_{\mathcal{B}}(0)$ (bosonic)	$\zeta'_{\mathcal{B}}(0)$ (bosonic)	$\zeta'_{\Delta_2}(0)$ (fermionic)
Linear coefficient a	8.528	3.564	$\sqrt{5}$
Quadratic coefficient C	$-\zeta(3)$	$+\zeta(3)/12$	$-\frac{\zeta(3)}{8}(1+\frac{\zeta(3)}{28})$
Torsion exponent σ	(absorbed)	$\zeta(3)/(16\pi^2)$	$\zeta(3)/(8\pi^2)$
Multiplicity per gen.	1	2	1
Splitting mechanism	none	$(2,0)/(1,1)$	none

JLN Supporting equations:
eq. 400-428 6.20 Neutrino Masses

eq . 400

In[6248]:=

r5Cliff = 1/√3 ;

γ5eff = N5 / r5Cliff

Out[6249]=

8√3π³

eq . 410-414

In[6250]:=

λk := (#+2) (#+6) &;

λk /@ Range@3

Out[6251]=

{21, 32, 45}

In[6252]:=

6!

Out[6252]=

720

In[6253]:=

dk =
$$\frac{\# \left(\# + 1\right) \left(\# + 3\right) \left(\# + 4\right)^2 \left(\# + 5\right) \left(\# + 7\right) \left(\# + 8\right)}{6 !}$$
&;

dk /@ Range@3

Out[6254]=

{120, 945, 4312}

eq . 415-416

In[6255]:=

Clear@ξΔ2p0;

$$\xi_{\Delta 2 p 0}=-\sum_{k=1}^2\left(d k @ k\left(\operatorname{Log}[k+2]+\operatorname{Log}[k+6]\right)\right) /. s 1 \operatorname{Rep}$$

N@%

Out[6256]=

−120 (Log[3] + Log[7]) − 945 (Log[4] + Log[8])

Out[6257]=

−3640.46

In[6258]:=

Clear@ξΔ2p0;

In[6259]:=

19 = 16 ;

Λ9 = 32π⁵;

$$k 9=\frac{1}{\Lambda 9} \operatorname{e}^{\xi_{\Delta 2 p 0} / 19} /. s 1 \operatorname{Rep}$$

inN@%

Out[6261]=

e^{ξΔ2p0/16}

Out[6262]=

$$\frac{\operatorname{e}^{\xi_{\Delta 2 p 0} / 16}}{32 \pi^5}$$

0.000102118 × 2.71828^{0.0625 ξΔ2p0}

In[6263]:=

$\xi_{\Delta 2 p 0}=-.41364;$

k_9

Out[6264]=

0.0000995115

In[6265]:=

Clear@r9Cliff;

In[6266]:=

Off[Solve::ratnz]

In[6267]:=

$a_9:=\frac{k_9 \, \mathcal{N}_9}{r_9 \text{Cliff}} /. \text{slRep};$

Solve[a9 == $\sqrt{5} \, e^{\xi_{\Delta 2 p 0}/19}$]][1, 1]

r9Cliff = Values@%

Out[6268]=

$r_9 \text{Cliff} \rightarrow 0.447214$

Out[6269]=

0.447214

In[6270]:=

$\Delta_9=\sqrt{2 \, \pi} \, v \, k_9^4$

N[$\Delta_9 /. \text{cRep}$]

convert[%, Mega eVperC2]

Out[6270]=

$$\frac{2.30525 \times 10^{-16} \text{MassUnit} \sqrt{2 \, \pi + \alpha}}{\alpha^4}$$

Out[6271]=

$2.03891 \times 10^{-7} \text{MassUnit}$

Out[6272]=

$6.05027 \times 10^{-11} \text{eVperC2 Mega}$

In[6273]:=

;

This is exponentially close to unity (the exponent is $\zeta'_{\Delta_2}(0)/16 \approx -0.026$). The remaining geometric factor is the reciprocal Clifford torus radius on $S^9 \subset \mathbb{C}^5$:

$$r_{\text{Cliff}}^{(9)}=\frac{1}{\sqrt{5}},$$

In[6274]:=

$\sqrt{5}.$

Out[6274]=

2.23607

In[6275]:=

a9

Out[6275]=

2.179

eq . 417-421

In[6276]:=

$$c_9=-\frac{\xi_{@3}}{8} \left(1+\frac{\xi_{@3}}{28}\right) /. \text{slRep};$$

inN@%

Out[6277]=

$$-\frac{1}{8} \left(1+\frac{\text{Zeta}[3]}{28}\right) \text{Zeta}[3]$$

-0.156708

In[6278]:=

$\beta_9=\beta_5;$

inN@%

Out[6279]=

$$\frac{\text{Zeta}[5]}{8 \, \pi^4}$$

0.00133064

In[6280]:=

$\sigma_9=\sigma_5;$

inN@%

Out[6281]=

$$\frac{\text{Zeta}[3]}{16 \, \pi^2}$$

0.00761211

eq . 409

In[6282]:=

$(\star \vee \star)$

$$m_{\nu @ n_-}:=\Delta_9 \, (n+1) \, e^{\left(a_9 \, n + c_9 \, n^2 + \beta_9 \, \frac{n \, (n+1)}{2} + \sigma_9 \, \text{Log}[\tau_{K @ n}]\right)};$$

{#, convert[mv@#, eVperC2]} & /@ Range@3 // MatrixForm

Out[6283]//MatrixForm=

$$\begin{pmatrix} 1 & 0.000915489 \text{eVperC2} \\ 2 & 0.00768492 \text{eVperC2} \\ 3 & 0.0414383 \text{eVperC2} \end{pmatrix}$$

CKM/PMNS Matrices:

JLN reference snapshots:

In[6284]:=

Summary of Mixing Predictions			
Observable	Our prediction	PDG value 78	Status
$ V_{us} $ (Cabibbo)	0.2233	0.2245 ± 0.0008	-1.5σ
$ V_{cb} $	0.0426	0.0421 ± 0.0008	$+0.6\sigma$
$ V_{ub} $	(phase-dependent)	0.00382 ± 0.00024	Structural mechanism identified
CKM CP violation	Nonzero	$J = (3.08 \pm 0.15) \times 10^{-5}$	Follows from $\lambda_T(k)$
PMNS: large angles	Yes	$\theta_{12} = 33.4^\circ$	Structural (mild ν hierarchy)

The Cabibbo angle and $|V_{cb}|$ are genuine zero-parameter predictions within the PDG uncertainty bands. The 2/3 orbifold restriction factor entering $|V_{cb}|$ is not fitted but forced by the Seifert structure of the trefoil complement—the same geometric object that determines the first-generation quark doublet ratio. The structural predictions (tridiagonal texture, CKM–PMNS hierarchy contrast, CP violation from generation-dependent λ_T) are established from the spectral geometry of the Hopf shell hierarchy.

In[6296]:=

The element $|V_{ub}|$ involves the full complex phase structure from $\lambda_T(n)$ generation-dependence. At leading order in the hierarchical expansion, $|V_{ub}| \approx |V_{us}| \cdot |V_{cb}| \cdot F(\delta\phi)$, where the phase-dependent function F is bounded by $0 \leq F \leq 1$ and encodes the CP-violating interference between the up-type and down-type contributions to the 1-3 element. The PDG value $|V_{ub}| = 0.00382 \pm 0.00024$ requires $F \approx 0.40$, corresponding to a specific value of the relative holonomy phase computable from the $\lambda_T(n)$ differences.

JLN Supporting equations:

eq . 435-444

In[6285]:=

$\phi \mathbf{k} = 2 \, \pi \, / \, 3;$

In[6286]:=

$\zeta_{12} := \sqrt{\mathbf{m} @ \# \times \mathbf{m} [\# + 1]} \, / . \, \mathbf{slRep} \, / . \, \Delta \mathbf{Hopf} \rightarrow 1 \, \&;$

$\zeta_{12} @ 2 \, / . \, \mathbf{cRep} \alpha$

$\mathbf{NRv1} @ \% \, / . \, \mathbf{cRep}$

Out[6287]=

$2 \, \sqrt{3} \, \sqrt{\mathbf{e}^{\frac{27.0342 + \frac{5 \alpha}{6}}{}}}$

Out[6288]//NumberForm=

$2.57819017262330 \times 10^6$

In[6289]:=

$\mathbf{dim} \Omega_{11} = 4;$

$\mathbf{dim} \Omega_{20} = 6;$

$\mathbf{R23} = \mathbf{dim} \Omega_{11} \, / \, \mathbf{dim} \Omega_{20}$

Out[6291]=

$\frac{2}{3}$

JGM Symbolic Analysis:

Vus & Vcb

In[6292]:=

$\mathbf{Vus} = \sqrt{\frac{\mathbf{muds} [2, \mathbf{False}]}{\mathbf{muds} [3, \mathbf{False}]}}$

$\mathbf{NRv1} \left[\mathbf{VusN} = \sqrt{\frac{\mathbf{muds} @ 2}{\mathbf{muds} @ 3}} \right]$

Out[6292]=

$1. \sqrt{\frac{\frac{\exp \left(\frac{\zeta (3)}{12} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) + \frac{\zeta (5)}{8 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{2}{3 \sqrt{3}} \right)}{192 \sqrt{3} \pi^8}}{\frac{\sqrt{3} \exp \left(\frac{\zeta (3)}{3} + 2 \left(-\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) - \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{8 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (4)}{16 \pi^2} \right)}{256 \pi^8}}}$

Out[6293]//NumberForm=

0.22327018340681

In[6294]:=

$\mathbf{Vcb} = \frac{2}{3} \mathbf{Abs} \left[\sqrt{\frac{\mathbf{muds} [3, \mathbf{False}]}{\mathbf{mcbt} [2, \mathbf{False}]}} - \sqrt{\frac{\mathbf{mcbt} [1, \mathbf{False}]}{\mathbf{mcbt} [3, \mathbf{False}]}} \right] \, / . \, \mathbf{slRep}$

$\mathbf{NRv1} \left[\mathbf{VcbN} = \frac{2}{3} \mathbf{Abs} \left[\sqrt{\frac{\mathbf{muds} @ 3}{\mathbf{mcbt} @ 2}} - \sqrt{\frac{\mathbf{mcbt} @ 1}{\mathbf{mcbt} @ 3}} \right] \right]$

Out[6294]=

$\frac{2}{3} \mathbf{Abs} \left[-1. \sqrt{\frac{\frac{\sqrt{3} \exp \left(\frac{\zeta (3)}{3} + 2 \left(\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) + \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{8 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (4)}{16 \pi^2} \right)}{256 \pi^8}}{\frac{\exp \left(\frac{3 \zeta (3)}{4} + 3 \left(-\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) + \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{4 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (3)}{16 \pi^2} \right)}{64 \sqrt{3} \pi^8}}} + 1. \sqrt{\frac{\frac{\sqrt{3} \exp \left(\frac{\zeta (3)}{3} + 2 \left(-\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) - \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{8 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (4)}{16 \pi^2} \right)}{256 \pi^8}}{\frac{\exp \left(\frac{3 \zeta (3)}{4} + 3 \left(\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) - \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{4 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (3)}{16 \pi^2} \right)}{64 \sqrt{3} \pi^8}}} \right]$

Out[6295]//NumberForm=

0.04263152122963

Vub with $F\zeta\phi$ evaluated

In[6297]:=

$\mathbf{F}\zeta\phi = 0.40;$

In[6298]:=

$\mathbf{Vub} = \mathbf{Abs} @ \mathbf{Vus} \times \mathbf{Abs} @ \mathbf{Vcb} \, \mathbf{F}\zeta\phi \, / . \, \mathbf{slRep}$

$\mathbf{VubN} = \mathbf{Abs} @ \mathbf{VusN} \times \mathbf{Abs} @ \mathbf{VcbN} \, \mathbf{F}\zeta\phi \, / . \, \mathbf{slRep}$

Out[6298]=

$0.266667 \sqrt{\mathbf{Abs} \left[\frac{\frac{\exp \left(\frac{\zeta (3)}{12} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) + \frac{\zeta (5)}{8 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{2}{3 \sqrt{3}} \right)}{192 \sqrt{3} \pi^8}}{\frac{\sqrt{3} \exp \left(\frac{\zeta (3)}{3} + 2 \left(-\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) - \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{8 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (4)}{16 \pi^2} \right)}{256 \pi^8}}} \right]}$

$\mathbf{Abs} \left[-1. \sqrt{\frac{\frac{\sqrt{3} \exp \left(\frac{\zeta (3)}{3} + 2 \left(\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) + \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{8 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (4)}{16 \pi^2} \right)}{256 \pi^8}}{\frac{\exp \left(\frac{3 \zeta (3)}{4} + 3 \left(-\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) + \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{4 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (3)}{16 \pi^2} \right)}{64 \sqrt{3} \pi^8}}} + 1. \sqrt{\frac{\frac{\sqrt{3} \exp \left(\frac{\zeta (3)}{3} + 2 \left(-\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) - \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{8 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (4)}{16 \pi^2} \right)}{256 \pi^8}}{\frac{\exp \left(\frac{3 \zeta (3)}{4} + 3 \left(\frac{\zeta (3)}{24 \pi} + \sqrt{3} \, \mathbf{e}^{\frac{1}{6} \left(\frac{5 \zeta (3)}{8 \pi^2} + \frac{3 \zeta (5)}{8 \pi^4} \right)} \left(\frac{\zeta (3)}{4 \pi^2} + 2 \right) - \frac{2}{\pi} \right) + \frac{3 \zeta (5)}{4 \pi^4} + \frac{5 \pi^2 \zeta (3) + 3 \zeta (5)}{16 \pi^4} + \frac{\zeta (3) \log (3)}{16 \pi^2} \right)}{64 \sqrt{3} \pi^8}}} \right]$

Out[6299]=

0.00380734

Magnetic Moments

Leptonic anomalous magnetic moments ($a_{e,\mu,\tau}$)

and gyromagnetic ratios ($g_{e,\mu,\tau}$):

JLN reference snapshots:

In[6300]:=

6.24.1 The Magnetic Moment as a Torsion Invariant

The contorsion 1-form of the Hopf fiber is $K_{U(1)} = \alpha d\theta/(2\pi)$, where θ parametrizes the S^1 fiber and α is the coupling strength derived from the spectral geometry of S^9 (Theorem 106). The magnetic moment of the n -th generation lepton is the ratio of the torsion-dressed fiber phase to the bare geometric phase:

$$\frac{g_n}{2} = \frac{2\pi + \Delta\phi_{\text{torsion}}(n)}{2\pi}, \tag{429}$$

6.24.3 Mass-Dependent Holonomy

A lepton heavier than the electron traverses a helical orbit on S^3 that deviates from the Reeb flow. The deviation accumulates additional fiber holonomy proportional to $\mathcal{L} = \ln(m_n/m_e)$. For the electron ($\mathcal{L} = 0$) these terms vanish.

Three contributions arise from the interaction of the helical orbit with the fiber torsion:

(i) **Helical holonomy.** The helical orbit sweeps area $\propto \mathcal{L}^2$ on the base $\mathbb{CP}^1$, reduced by $2\mathcal{L}$ from the torsion back-reaction on the geodesic deviation. The contact normalization $\mathcal{N}_3/(8\ell) = \pi/12$ sets the scale. The fiber torsion dresses the holonomy coefficient by the factor $(1 - \alpha\zeta(3)/(4\pi\ell))$, the same S^3 torsion attenuation that enters the universal phase:

$$\Delta\phi_4 = \left(\frac{\alpha}{2\pi}\right)^2 \frac{\pi}{12} \left(1 - \frac{\alpha\zeta(3)}{4\pi\ell}\right) \mathcal{L}(\mathcal{L}-2).$$

(430)

217

The universal torsion phase, dressed by all four shells of the Hopf hierarchy, is

$$\Delta\varphi_{\text{univ}} = \alpha \exp\left(-\frac{\alpha\zeta(3)(2\ell+1)}{4\pi\ell} - \frac{\alpha^2\zeta(5)}{4\pi^2} - \frac{\alpha^3|\zeta'_{\mathcal{B}_7}(0)|}{\ell_7} - \frac{\alpha^4|\zeta'_{\Delta_2}(0)|}{\ell_9}\right).$$

(431)

(ii) **Spectral determinant correction.** The winding-sector spectral zeta $\zeta_{\mathcal{B}}(0) = 1/2$ (the regularized mode count) and the summed torsion exponent $4\sigma_3$ correct the holonomy:

$$\Delta\phi_5 = \left(\frac{\alpha}{2\pi}\right)^3 \mathcal{C}_{\text{det}} \mathcal{L}, \quad \mathcal{C}_{\text{det}} = -(\tfrac{1}{2} - 4\sigma_3).$$

(432)

(iii) **Holonomy trace.** The helical holonomy propagates through the S^3 spectral geometry, with the complementary projection $(1 - \sigma_3)$ —the fraction of the spectral determinant not entering the mass spectrum—dressing the second iteration:

$$\Delta\phi_6 = -\left(\frac{\alpha}{2\pi}\right)^4 (1 - \sigma_3) \mathcal{L}^2(\mathcal{L}-2).$$

(433)

6.24.4 The Complete Magnetic Moment

$$\frac{g_n}{2} = 1 + \frac{1}{2\pi} \Delta\phi_{\text{univ}} + \Delta\phi_4 + \Delta\phi_5 + \Delta\phi_6,$$

(434)

and $\Delta\phi_{4,5,6}$ from the helical holonomy, spectral determinant correction (432), and holonomy trace (433) terms derived in Section 6.24.3

Every quantity appearing in (434) is a spectral invariant of the Hopf bundle derived elsewhere in this paper. The magnetic moment is not computed from Feynman diagrams, perturbation theory, or lattice simulations. It is the torsion of the $U(1)$ fiber—the same torsion that generates the mass spectrum, the gravitational constant, and the dark sector—evaluated at the lepton’s Beltrami eigenmode.

6.24.5 Predictions

	Topological Prediction	Lattice QCD[81]	PDG[78]	Within PDG error?	Beats LQCD?
a_e	$1.159\,652\,180 \times 10^{-3}$	—	$1.159\,652\,181(13) \times 10^{-3}$	Yes (0.08σ)	—
a_μ	$1.165\,920\,747 \times 10^{-3}$	$1.165\,920\,33(62) \times 10^{-3}$	$1.165\,920\,715(146) \times 10^{-3}$	Yes (0.22σ)	Yes ($12\times$)
a_τ	$1.177\,365 \times 10^{-3}$	—	—	True prediction	

The lattice QCD value for the muon is the 2025 White Paper (WP25) result [81], which deviates from experiment by $2.6\sigma_{\text{exp}}$ with theoretical uncertainty $\pm 6.2 \times 10^{-10}$. The present theory deviates by $0.22\sigma_{\text{exp}}$ with zero free parameters—12 times closer to experiment than lattice QCD and 107 times closer than the 2020 dispersive determination [82]. The universal phase receives four shell dressings (S^3 , S^5 , S^7 , S^9), with the S^7 contribution—the topological counterpart of hadronic vacuum polarization—entering at third order in α with framing $\ell_7 = 56 = \binom{8}{3}$. The tau prediction $a_\tau^{\text{topological}} = 1.177\,365 \times 10^{-3}$ is a true *a priori* prediction with no existing measurement at this precision; Belle II [83] and CLIC [84] will reach the required sensitivity, providing a direct falsification channel.

JLN Supporting equations:

eq . 431

In[6302]:=

$$\Delta\phi_{\text{UnivExp}} = -\frac{\alpha}{2\pi} \left\{ \frac{\zeta@3 (2\,1+1)}{2\,1}, \frac{\alpha}{2\pi} \zeta@5, \alpha^2 \frac{\zeta\Delta2p0}{19} \right\} /. \text{slRep}$$

% /. cRep

Total@%

1 / %

Out[6302]=

$$\left\{ -\frac{13\,\alpha\,\text{Zeta}[3]}{24\,\pi}, -\frac{\alpha^2\,\text{Zeta}[5]}{4\,\pi^2}, 0.00411455\,\alpha^3 \right\}$$

Out[6303]=

$$\left\{ -0.00151242, -1.39868 \times 10^{-6}, 1.59889 \times 10^{-9} \right\}$$

Out[6304]=

$$-0.00151382$$

Out[6305]=

$$-660.582$$

In[6306]:=

$$\mathbf{e}^{\text{Total@}\Delta\phi_{\text{UnivExp}}}$$

% /. cRep

1 / %

Out[6306]=

$$\mathbf{e}^{0.00411455\,\alpha^3 - \frac{13\,\alpha\,\text{Zeta}[3]}{24\,\pi} - \frac{\alpha^2\,\text{Zeta}[5]}{4\,\pi^2}}$$

Out[6307]=

$$0.998487$$

Out[6308]=

$$1.00151$$


```
In[6309]:=
ΔφUniv = α eTotal@ΔφUnivExp /. slRep
% /. cRep
1 / %
Out[6309]=
0.00411455 α3 - 13 α Zeta[3]24 π - α2 Zeta[5]4 π2 α
Out[6310]=
0.00728631
Out[6311]=
137.244
eq . 430-433
In[6312]:=
(* This equation is at odds with the Weinberg angle reference above in eq. 373 as 4π2 *)
In[6313]:=
Clear@N3;
```

Solve $\left[\frac{N3}{81} == \frac{\pi^{(*)1**}2^{(**)}}{12} \right] /. slRep$ [[1, 1]]

```
N3 = Values@%
Out[6314]=
N3 -> 4 π2
Out[6315]=
4 π2
In[6316]:=
L = Log[m[# - 3] / m@1] /. slRep &;
aStr = {"a_e", "a_μ", "a_τ"};
{#, aStr[[# - 3]], L@#, L@# /. cRep} /. meMeVso1 & /@ Range[4, 6] // TableForm
Out[6318]//TableForm=
4      a_e      0.      0.
5      a_μ      Log $\left[\frac{3}{2} e^{4.92492 + \frac{\alpha}{6}}\right]$       5.3316
6      a_τ      Log $\left[2 e^{7.45841 + \frac{\alpha}{3}}\right]$       8.15399
In[6319]:=
```

Δφ@4 := $\left(\frac{\alpha}{2\pi}\right)^{4-2} \frac{N3}{81} \left(1 - \frac{\alpha \xi@3}{41}\right) \mathcal{L}@4 (\mathcal{L}@4 - 2) /. slRep$;

```
In[6320]:=
cDet = 4 σ3 - 1 / 2;
Δφ@5 :=  $\left(\frac{\alpha}{2\pi}\right)^{5-2} cDet \mathcal{L}@5 /. slRep$ ;
```

Δφ@6 := $\left(\frac{\alpha}{2\pi}\right)^{6-2} (1 - \sigma3) \mathcal{L}[6]^2 (\mathcal{L}@6 - 2) /. slRep$;

```
In[6323]:=
{"Δ" <> ToString@#, res = Δφ@#, res /. cRep} & /@ Range[4, 6] // TableForm
```

Δ4	0.	0.
Δ5	$\frac{\alpha^3 \operatorname{Log}\left[\frac{3}{2} e^{4.92492 + \frac{\alpha}{6}}\right] \left(-\frac{1}{2} + \frac{\operatorname{Zeta}[3]}{\pi^2}\right)}{8 \pi^3}$	-3.15895×10^{-9}
Δ6	$\frac{\alpha^4 \left(-2 + \operatorname{Log}\left[2 e^{7.45841 + \frac{\alpha}{3}}\right]\right) \operatorname{Log}\left[2 e^{7.45841 + \frac{\alpha}{3}}\right]^2 \left(1 - \frac{\operatorname{Zeta}[3]}{4 \pi^2}\right)}{16 \pi^4}$	7.21789×10^{-10}

JGM Symbolic Analysis:

```
a_e, μ, τ
In[6324]:=
ae
Out[6324]=
0.001159652180
In[6325]:=
NRv1[aμ = 2 ae]
NRv1[aτ =  $\frac{9}{4}$  ae]
Out[6325]//NumberForm=
0.00231930436092
Out[6326]//NumberForm=
0.00260921740604
```

```
g_e, μ, τ
In[6327]:=
NRv1@ $\frac{\alpha}{2\pi} /. cRep$ 
Out[6327]//NumberForm=
0.00116140973209
In[6328]:=
NRv1[ge = -2 (1 + ae) ]
Out[6328]//NumberForm=
-2.00231930436092
eq . 434
In[6329]:=
```

gn = -2 $\left(1 + \frac{\Delta\phi\text{Univ}}{2\pi} + \Delta\phi@4 + \Delta\phi@5 + \Delta\phi@6\right) /. slRep$

NRv1[% /. cRep]

```
Out[6329]=
-2  $\left(1. + \frac{e^{0.00411455 \alpha^3 - \frac{13 \alpha \operatorname{Zeta}[3]}{24 \pi} - \frac{\alpha^2 \operatorname{Zeta}[5]}{4 \pi^2}} \alpha}{2 \pi} + \frac{\alpha^4 \left(-2 + \operatorname{Log}\left[2 e^{7.45841 + \frac{\alpha}{3}}\right]\right) \operatorname{Log}\left[2 e^{7.45841 + \frac{\alpha}{3}}\right]^2 \left(1 - \frac{\operatorname{Zeta}[3]}{4 \pi^2}\right)}{16 \pi^4} + \frac{\alpha^3 \operatorname{Log}\left[\frac{3}{2} e^{4.92492 + \frac{\alpha}{6}}\right] \left(-\frac{1}{2} + \frac{\operatorname{Zeta}[3]}{\pi^2}\right)}{8 \pi^3}\right)$ 
Out[6330]//NumberForm=
-2.00231930092480
In[6331]:=
```

```
NRv1@ElectronGFactor
NRv1@ElectronGFactorError
Out[6331]//NumberForm=
-2.00231930436092
Out[6332]//NumberForm=
3.6000000000000000 × 10-13
In[6333]:=
NRv1[%% % / % /. slRep]
Out[6333]//NumberForm=
0.99999999828393
In[6334]:=
NRv1@MuonGFactor
NRv1@MuonGFactorError
Out[6334]//NumberForm=
-2.00233184123000
Out[6335]//NumberForm=
8.2000000000000000 × 10-10
```

```
In[6336]:=
doStdErr["gn", "ElectronGFactor", 1, 1]
gn=-2.00232
SI Std. Error = 3.6×10-13
New/Old Std. Error = 1.71607×10-9
Std. Error Factor (<1 is inside Experimental Error) = 9541.35
```

Supplemental Theoretical Analysis and Information

H4 120 - cell Hopf fibration

How to get to 16D from SO(10)?

Clearly acknowledging the "overgroup" of E_8 (or at least E_6 here):

Numerator is SO(14) (rank 7 not 6) and Denominator is rank 1 SO(2),

SO $\mathfrak{g}_{\text{Total}}$ =SO(12) NOT SO(10) and rank 6

Hodge Star (★) dimensional reduction (as used in octonionic STA) :

